A comparison of multicontinuum theory based failure simulation with experimental results

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Abstract

This paper is part of a broader exercise to determine the predictive capability of several currently accepted composite failure criteria. Blind failure and stress–strain predictions were originally submitted by us to the organizers of the exercise for 14 different cases involving four different composite materials. A finite element based analysis was used to simulate progressive failure of the various composite laminates under uniaxial and multiaxial loading. Under the assumption of the composite acting as a multi-continuum, fiber and matrix constituent stress and strain states were derived. This information formed the input to a quadratic, stress-interactive constituent-based failure criteria to predict damage at the composite level. Two-dimensional failure envelopes and stress–strain curves for the composite were developed. In what follows, the experimental results have been made available to us for comparison against our analytical failure simulations. Generally, these analytical predictions were in good agreement with experimental results. The presence of simultaneous normal and shear stresses in a composite laminate was found to significantly enhance shear strength which was not accounted for in the proposed failure criteria.

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1. Introduction

Ultimately, the goal of a failure prediction methodology is to extrapolate results of a relatively easy-to-conduct laboratory experimental program, utilizing simple test specimen geometry and uniaxial loading, to real-world applications with complex structural geometry and multiaxial loading. Typical failure criteria are semi-empirical, phenomenological models attempting to describe experimental observations. Generally, only the occurrence of failure is predicted, not all possible modes of failure. For instance, in the case of continuous fiber composite materials, failure details such as fiber pullout, fiber breakage, fiber micro-buckling, matrix cracking, delamination, etc., are not discerned.

In addition to multiple failure modes, failure of a composite material is further complicated by the interplay of load transfer between the constituents. The approach to composite failure analysis presented herein utilizes a multicontinuum theory (MCT) to illuminate this interplay by decomposing the composite stress and strain fields into those of its constituents.

Hashin [1] states that most failure criteria that account for stress interaction are quadratic in stress and that choice of a quadratic polynomial is based solely on curve fitting considerations. A quadratic approximation, he notes, “is the simplest presentation which can fit the data reasonably well and in view of the significant scatter of failure test data it hardly seems worthwhile to employ cubic or higher approximations.” MCT uses a quadratic, stress-interactive failure criteria. However, in a major departure from traditional failure analyses, MCT applies the failure criteria at the constituent rather than the composite scale.

The current MCT research is applied to composite laminates consisting of unidirectional continuous fiber lamina. Material symmetry considerations allow us to assume transverse isotropy for each of the unidirectional lamina. Therefore, the constituent failure criteria may be expressed in terms of transversely isotropic stress invariants [2] given by:
Constituent failure parameters, as well as constituent elastic constants, are extracted from experimentally determined composite results with the aid of finite-element micromechanics and MCT analyses [4] and are thus in situ rather than bulk values. We also note that thermal residual stresses were neglected in all analyses.

2. Simulation versus experimental results

We are generally pleased with our failure simulations involving four different composite materials and fourteen cases provided by the organizers of the failure exercise [5]. Generally, our results are conservative, i.e., falling inside the experimentally generated failure envelopes. We did not require any information other than that provided by the organizers, and the initial failure simulations were conducted by a single individual in less than 80 h.

As in so many research endeavors, the experimental data raise as many questions as they answer. Questions such as: (1) the validity of extrapolating unidirectional lamina behavior to lamina that are part of a general laminate (cases 1, 3, and 3). (2) The effect of simultaneous shear and normal lamina stresses on lamina strength (cases 1, 2, 4, 5, and 9). (3) The constraining effects of adjacent lamina, in particular orthogonal orientations, on lamina strength (cases 10, 10, and 14).

In the following sections, we compare our failure simulations with the experimentally generated data for each of the 14 cases and briefly discuss the correlation between them. We also correct our analyses for cases 9, 11, and 13 previously presented in Part A of the failure exercise.

2.1. Case 1

The analytically generated biaxial, \( \sigma_{12}, \tau_{xy} \), failure envelope for a \( 0^\circ \) E-Glass/LY556/HT907/DY063 lamina, shown in Fig. 1, was in good agreement with the experimental data. There is no doubt that the correlation would improve significantly if the compressive unidirectional strength, provided by the organizers, was in better agreement with the case 1 data set. This difference may be the result of using material properties derived from tests of finite-width test coupons that are influenced by free-edge effects which are absent in tubular specimens [6]. The net result of increasing the unidirectional lamina compressive strength value in the failure criteria would be to move the analytical failure envelope in quadrant II outward away from the axis origin and in better correlation with the experimental data.

We also note that in this and other cases, most notably 2 and 9, lamina in situ shear strength appears to increase in the presence of simultaneous normal stress.
The absence of an explicit shear/normal interaction term in the proposed MCT failure criteria may need to be addressed. There is an implicit normal-shear interaction term in the full failure criteria originally proposed by us [3, 4]. A truncated form of the full criteria which eliminated the implicit normal-shear term was used for the analyses conducted as part of this exercise because of the absence of material test data necessary to determine this term.

2.2. Case 2

The analytical biaxial, \( \sigma_{x}, \tau_{xy} \), final failure envelope for a 0° lamina made of CFRP T300/914C material, shown in Fig. 2, is again conservative without being overly so. As in case 1, the shear strength of the material appears to be enhanced in the presence of both tensile and compressive normal stresses. A question arises concerning this behavior as to whether it is peculiar to single UD lamina, i.e., does it also occur within a laminate of a general stacking sequence? In the single case (5) presented by the organizers of a general laminate under \( \sigma_{x}, \tau_{xy} \) loading, there is some evidence of increasing shear failure stress in the presence of moderate axial tensile and compressive stress. In future research, we intend to conduct additional analyses and experiments to further investigate the effect of normal-shear interaction on laminate strength and the necessity of accounting for it in the analysis of structural laminates. Should it be proven to be a significant effect, it would provide the motivation for adding an explicit shear-normal stress interaction term to our failure criteria.

2.3. Case 3

The failure envelope shown in Fig. 3 is for a biaxial, \( \sigma_{x}, \sigma_{y} \), loading of a 0° lamina of E-glass/MY750 epoxy. Data necessary to develop a full experimental failure envelope in all four quadrants are lacking. Hence, only partial conclusions can be drawn concerning correlation between analysis and experimental results. MCT is in good correlation with the data in quadrant I and not so in quadrant IV.

The rectangular failure envelope predicted by MCT is attributed to a distinct transition from fiber to matrix failure and is similar to one that would be produced by a simple maximum stress failure criterion \( \text{max} \left( \frac{\sigma_{yy}}{S_{yy}}, \frac{\sigma_{xx}}{S_{xx}} \right) \geq 1 \). The distinct transition from matrix to fiber failure is a result of assuming that the strength of the fiber constituent controls composite failure in the direction parallel to the fiber and the matrix strength controls failure in the transverse direction. The unidirectional (UD) strength data provided by the organizers tends to support this assumption in the tension-tension regime where there is a two order of magnitude difference in the lamina’s longitudinal \( \left( +S_{11} = 1280 \text{ MPa} \right) \) and transverse strengths \( \left( +S_{22} = 40 \text{ MPa} \right) \). One might assume that this effect would also hold in the tension-compression regime since there is still an order of magnitude difference in the...
lamina’s longitudinal tensile ($+S_{11}=1280$ MPa) and transverse compressive ($-S_{22}=145$ MPa) strengths.

The question is raised; is the experimental behavior in quadrant IV peculiar to a single UD lamina or does it also occur within a laminate of general stacking sequence? If further experiments indicated that the stress interaction in the $+\sigma_y - \sigma_x$ regime were important in the analysis of general structural laminates, it may be necessary to add a normal–normal stress interaction term to increase the accuracy of failure predictions for this material.

2.4. Case 4

We are very pleased with the correlation between analytical and experimental $\sigma_x - \sigma_y$ failure envelopes, shown in Fig. 4, for the (90°/±30°) E-glass/LY556 epoxy laminate. This case represents, to us, a general structural laminate under in-service loadings. Again, our results are generally conservative. The decrease in correlation in quadrant III, compression-compression loading, is likely due to buckling of the test specimen as noted by the organizers [7]. Buckling is not accounted for in the current MCT failure criteria. We note also that failure points A, D, E, F, and G were all compression induced and would be expanded outward if higher UD lamina compression values were used as indicated by case 1. In the region about point C, longitudinal (along the fiber) tension and shear coexist in the ±30° lamina which ultimately causes laminate failure. But a shear strengthening effect, as seen in the single lamina test of case 1, does not appear to exist since MCT tends to slightly over-predict the ultimate laminate strength. Any discussion by the authors on the shear strengthening effect at this point would be pure speculation. As we stated previously, we were not aware of any shear/normal stress interaction on lamina failure before this exercise and we intend to conduct additional analyses and experiments to further investigate this phenomena.

2.5. Case 5

We are also pleased with the correlation between analytical and experimental failure envelopes, shown in Fig. 5, for the (90°/±30°) E-glass/LY556 epoxy laminate under combined $\sigma_x - \tau_{xy}$ stresses. Again, our results are generally conservative. Because compression was the major contributor to failure at points A, B, and C, increasing the UD lamina compression strength would result in an improved analytical versus experimental correlation in the shear-compression region of quadrant II. As noted in case 1, there appears to be justification for increasing this strength.

We note that in the region about points E, F, G, and H, longitudinal tension and shear stresses coexist and there is a lower correlation between analytical and experimental results. Likewise there is a lower analytical versus experimental correlation in the region about points A, B, and C where compression and shear stresses coexist. This information provides additional evidence that there may be a need to account for normal-shear stress interaction with an additional term in the MCT failure criteria.

2.6. Case 6

Excellent correlation is observed between the analytical and experimental $\sigma_x - \sigma_y$ failure envelopes in quadrants I and IV, shown in Fig. 6, for the AS4/3501-6, [0°/±45°/90°]S laminate. Again, we are pleased with the results here because this case represents a widely used
material and laminate configuration under realistic multiaxial loads. MCT appears to predict a particular flattening in the laminate failure envelope between points D and E but does so unconservatively. The reason for the overestimated laminate strength is not clear because the failure appears to be due to simple tension failure of the fiber constituent in the 0° lamina. The lack of correlation in quadrant III compression-compression loading is likely due to buckling of the test specimen, as noted by the organizers [7], which is not accounted for in the current MCT failure criteria. The MCT failure simulation also predicts non-catastrophic laminate damage, in quadrant I of Fig. 6, due to matrix tensile failure that correlates to results provided by the organizers [7]. Case 8 presents a stress–strain curve extracted from this data.

2.7. Case 7

The stress–strain curve for a AS4/3501-6, [0°/±45°/90°]_S laminate under uniaxial tension, \( \sigma_y/\sigma_x = 1:0 \), is shown in Fig. 7. MCT correlates exactly with the experimental results in the early part of the loading and conservatively under-predicts the ultimate laminate strength by 8%. Strain jumps in the analysis curve at approximately 250 MPa indicate initial laminate damage due to transverse matrix tensile failures in the 0° lamina. Intermediate damage, predicted around 400 MPa in the form of matrix failure in the \( \pm 45° \) lamina, was caused by combined shear and tensile stresses. The organizers note [7] that a decrease in the slope of the experimental stress–strain curve around 400 MPa indicated a form of initial failure in the actual laminate. MCT predictions correlate with this behavior but slightly over-predict the softening of the laminate due to failed matrix induced damage. The overly soft predicted response is attributed to the damage model used in MCT which assumes a simple on/off constituent damage state variable. When a constituent fails, all its moduli are immediately reduced to a near zero value and an alternate set of composite properties, derived from micromechanics [3], is used. Actual damage evolves much more slowly and gradually. A more realistic damage model could be implemented but in keeping with the design tool philosophy inherent in the formulation of MCT, the strain jumps help visualize the occurrence of damage.

2.8. Case 8

The stress–strain response for the AS4/3501-6, [0°/±45°/90°]_S laminate under \( \sigma_y/\sigma_x = 2:1 \) biaxial tension is shown in Fig. 8. The correlation with experimental data for the biaxial loading considered here is significantly better that the uniaxial stress–strain correlation shown in Fig. 7. MCT under-predicts ultimate laminate strength by 3% and failure strain values are virtually identical. The MCT simulation indicates that initial laminate damage occurs around 240 MPa due to transverse matrix tensile failures in the 0° lamina followed by matrix failure in the \( \pm 45° \) lamina around 350 MPa caused by combined shear and tensile stresses. Additional intermediate damage is predicted at approximately 460 MPa due to tensile matrix failure in the 90° lamina. Correspondingly, the organizers note [7] that
non-linear stress–strain behavior, possibly due to matrix cracking, occurs in the experimental stress–strain curve of unlined specimens around 480 MPa resulting in significant softening of the laminate. This intermediate damage was visually manifested in the unlined specimens by fluid leakage at the 480 MPa stress level while the specimens tested with a liner failed at an approximate stress level of 600 MPa. We believe that the lower failure of the unlined specimen is possibly due to the corrosive and erosive effects of a high pressure fluid leaking through cracks in the lamina matrix. While the lined specimens may have experienced identical matrix cracking, the liner prevented any fluid damage to the composite. MCT does not model any fluid-structure interactions.

2.9. Case 9

We erroneously truncated our originally submitted $\sigma_x, \sigma_y$ failure envelope presented for the E-glass/MY750/HY917/DY063 $[\pm 55^\circ]$S laminate by assuming that initial failure in quadrant I was also final failure. A new portion of the final failure envelope occurring in the tension-tension region has been added. Initial and final failure envelopes are virtually unchanged in quadrants II–IV. The corrected envelope is shown in Fig. 9. Revised results for the failure envelope are summarized in Table 1.

We missed the final failure in quadrant I because a complete matrix tensile failure throughout the laminate caused the finite element program to conduct several hundred equilibrium iterations, which we previously mistook for catastrophic laminate failure, before convergence allowed continued loading. Final laminate failure in this quadrant is due to fiber shear:tension failure.

The MCT results tend to provide conservative two-dimensional failure predictions in $\sigma_x, \sigma_y$ space and are in good agreement with the data in the neighborhood of the origin. MCT significantly under predicts the laminate failure strength at the positive and negative $\sigma_y$ extremes. Failure around these regions (points “E” and “I”) was a result of combined shear and compressive or tensile stresses respectively. Again it appears that a shear-normal stress interaction term may be needed to account for enhanced shear strength in the presence of a normal stress. MCT predictions of intermediate damage conservatively correlate with the experimental intermediate failures, in regions close to the origin, of unlined tubes in which matrix cracking caused oil leakage. MCT predictions do not correlate well with experimentally determined intermediate damage states at higher $\sigma_x$ stresses. The lack of correlation here leads us to believe that a shear-normal interaction term may be needed for both fiber and matrix constituents. The lack of correlation in the presence of shear-normal stress is further discussed in the context of the stress–strain curves presented in cases 10 and 11.

![Fig. 9. Biaxial, $\sigma_x, \sigma_y$ failure envelope for a $[\pm 55^\circ]$S laminate made from E-glass/MY750/HT917/DY063.](image)

![Table 1](image)
The organizers note that the specimens used in the compression-compression quadrant were made of thicker tubes. External pressures up to 200 MPa were needed to fracture the specimen which generated a radial stress component. Multicontinuum Theory is fully three-dimensional as are the proposed failure criteria with the single assumption of lamina transverse isotropy. Radial stresses due to external pressure would be accounted for if they had been modeled as part of the load. We chose not to conduct the more complex analysis which would result in a three-dimensional ($\sigma_x, \sigma_y, \sigma_z$ stress space) failure envelope.

2.10. Case 10

Non-linear shear behavior characterizes the stress–strain curves of the E-glass/MY750/HY917/DY063, [±55]s laminate under uniaxial load, $\sigma_x: \sigma_y = 1:0$, as shown in Fig. 10. MCT is in good agreement with the experimental data to about 230 MPa ($|\varepsilon_x| = 1.7\%$) where catastrophic laminate failure is predicted due primarily to fiber shear failure ($K_{sf} = 0.96$, $K_{if} = 0.04$). The lack of correlation between analysis and experimental data at higher stress levels is a direct result of the experimental data to which the failure criteria were fitted. In particular, shear strengths and nonlinear shear stress–strain relations were taken from the pure shear stress–strain curve provided by the organizers [5]. These data specify a composite shear strength of $S_{12} = 72$ MPa and ultimate engineering shear strain of $|\varepsilon_x| = 4\%$. In contrast, linear classical lamination theory [8] predicts lamina shear stress levels, for a [±55]s laminate, on the order of $\sigma_{12} = 200$ MPa at the experimental failure stress of $\sigma_y = 600$ MPa. Using the generalized plane strain transformation equation,

$$\frac{\gamma_{12}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2} \sin 2\theta + \frac{\gamma_{xy}}{2} \cos 2\theta,$$

the local lamina shear strain, $\gamma_{12}$, corresponding to laminate strains $|\varepsilon_x| = \varepsilon_y = 1.7\%$ and $\gamma_{xy} = 0$, is 3.1% which is close to the shear failure strain provided by the organizers. This result is expected since the MCT shear failure parameters were derived from these data. At failure ($\varepsilon_x = -10.93\%$, $\varepsilon_y = 8.78\%$) the experimental lamina shear strain, $\gamma_{12}$, is calculated to be approximately 18.5% which is over four times that provided by the organizers.

Our analysis of this case predicts simultaneous shear and tensile lamina stresses of the same order of magnitude in the local (material) coordinate systems. Therefore, enhanced shear strength in the presence of normal stresses is again a possible contributor to the higher observed strengths. In addition to the effect of simultaneous shear-normal stresses, we believe the constraining influence of the laminate stacking sequence contributes to enhanced stress–strain ultimate values that cannot be predicted by extrapolation of the UD shear data. In case 12 matrix tensile strength of a [0°/90°]s laminate is enhanced without the presence of simultaneous shear stress and in case 14 the lamina shear strength of a [±45°]s laminate is enhanced without the presence of simultaneous normal stress.

2.11. Case 11

Corresponding to the results presented in case 9, we originally presented a truncated stress–strain curve prediction for the E-glass/MY750/HY917/DY063 [±55]s laminate under $\sigma_x: \sigma_y = 2:1$. Our original submission was just the initial portion (out to 0.4% strain) of the corrected stress–strain curve presented in Fig. 11. Initial damage in the form of complete matrix tensile failure throughout the laminate caused the finite element program to conduct several hundred equilibrium iterations,

![Fig. 10. Uniaxial, $\sigma_x: \sigma_y = 1:0$, stress–strain curves for a [±55]s laminate made from E-glass/MY750/HT917/DY063.](image)

![Fig. 11. Uniaxial, $\sigma_x: \sigma_y = 2:1$, stress–strain curves for a [±55]s laminate made from E-glass/MY750/HT917/DY063.](image)
which we previously mistook for catastrophic laminate failure, before convergence allowed continued loading.

Matrix tensile failure is predicted in the laminate around \(\sigma_y = 120\) MPa which is manifest on the analytical stress–strain curve as a strain jump. A corresponding strain increase can be seen in the experimental \(\sigma_y - \epsilon_y\) curve but occurring at \(\sigma_y = 260\) MPa. Under-prediction of the stress level at which this intermediate damage occurs is again most likely due to the effect of enhanced shear strength in the presence of a normal stress which is not accounted for by the MCT failure criterion. Close examination of the initial portion of the stress–strain curves shows that MCT over-predicts the slope on both \(x\) and \(y\) stress–strain curves. This initial over-prediction carries into the post-damage region. We cannot rationalize the lack of correlation with the experimental stress–strain curves especially in light of the relatively good curve correlations in case 10 (\(\sigma_y:\sigma_x = 1:0\)) for the identical laminate and in cases 13 and 14 for biaxial loading (\(\sigma_y:\sigma_x = 1:1\) and \(\sigma_y:\sigma_x = 1:1\) respectively) for a similar ([\(\pm 45\]^S] laminate. The general dissimilarity between the nonlinear experimental and relatively linear analytical stress–strain curve is again the result of a simple on/off constituent damage model used by MCT whereas actual damage evolves in a more gradual and continuous manner.

The organizers note final fracture of the [\(\pm 55\]^S] laminates occurs at approximately \(\sigma_y = 740\) MPa which is higher than the levels of the experimental stress–strain graphs presented in Fig. 11. MCT over-predicts this laminate strength by approximately 11\% but is within 1\% of similar \(\sigma_y:\sigma_x = 2:1\) experimental results presented as part of case 9.

2.12. Case 12

The E-glass/MY750/HY917/DY063 stress–strain curves for a [0\(^\circ\)/90\(^\circ\)/0\(^\circ\)] laminate under uniaxial load, \(\sigma_y:\sigma_x = 1:0\), are shown in Fig. 12. There is excellent correlation between the analytical and experimental results. Initial laminate damage due to tensile matrix failure in the 0 lamina is predicted by MCT at approximately 80 MPa whereas the organizers note the onset of cracking at 117.5 MPa. Matrix tensile failure in the 90\(^\circ\) lamina is predicted by MCT at approximately 260 MPa where a corresponding cracking of the experimental specimen is noted at approximately 400 MPa. The lower predicted value may be due to transverse lamina strength being taken from uniaxial tests of unidirectional lamina (or laminates) in which matrix damage is catastrophic failure. Conversely, in a uniaxial (0\(^\circ\)) load test of a cross-ply there is a constraining effect of the 0\(^\circ\) lamina on the transverse failure of the 90\(^\circ\) lamina and vise-versa for a uniaxial 90\(^\circ\) load test. In situ properties for transverse lamina strength taken from a cross-ply test, and the MCT matrix strength parameter developed from it, would improve the analytical/experimental correlation for initial damage. The ultimate laminate strength for the [0\(^\circ\)/90\(^\circ\)/0\(^\circ\)] laminate was predicted within 2\% of the experimentally determined value.

The stress–strain curves for this laminate presented in Ref. [3] (Part A submission, Fig. 18), for the case where thermal stresses are considered, showed a peculiar behavior at small strain values. This behavior was a result of the authors’ attempt to illustrate the thermal analysis capability of MCT and to reflect the test data which were generated on laminates where thermal stresses are likely to have been developed as a result of curing at elevated temperature.

Our position regarding incorporating the effects of thermal stresses in a failure analysis is that our method of backing out constituent properties from experimental composite data mitigates the need to account for them in structural analysis or, at worst, reduces residual stress due to thermal mismatches to be secondary in their effect.

The peculiar behavior seen in the graph with thermal loads is readily explained by considering what we used as the reference (zero) strain state. Specifically, in a room temperature experimental test, there may be significant thermal stresses in the composite due to the curing process. Hence, as the experiment begins we have residual stresses at what is arbitrarily defined to be the zero strain state. The above situation is not what we modeled. Instead we defined our zero strain state prior to introducing the thermal load. If we were to subtract out the strains caused by the thermal loads, the resulting graph would be our predicted response for an experiment where we include the effect of the thermal residual stress. This would eliminate the peculiar behavior seen in Ref [3].

Fig. 12. Uniaxial, \(\sigma_y:\sigma_x = 1:0\), stress–strain curves for a [0\(^\circ\)/90\(^\circ\)/0\(^\circ\)] laminate made from E-glass/MY750/HT917/DY063.
In summary, for a direct comparison with experimental data, the curve we presented is not the most representative one. In retrospect, eliminating the thermal strains shown would have avoided any confusion interpreting the initial behavior of the laminate.

2.13. Case 13

We erroneously truncated our originally submitted stress–strain curves for a E-glass/MY750/HY917/DY063, [±45°]₅ laminate under a biaxial load, \(\sigma_y:\sigma_x = 1:1\). Our original submission was just the initial portion (extending to only 0.2% strain) of the corrected stress–strain curve shown in Fig. 13. There is excellent correlation between the analytical and experimental stress–strain curves. Initial laminate damage due to tensile matrix failure in all lamina is predicted by MCT at approximately 70 MPa which corresponds to the initial cracking noted by the organizers between 50 and 70 MPa. The laminate exhibits a near linear stress–strain response until tensile failure of the fibers is predicted at approximately 620 MPa. MCT over-predicts the ultimate laminate strength by approximately 24%.

2.14. Case 14

Stress–strain curves for an E-glass/MY750/HY917/DY063, [±45°]₅ laminate under biaxial load, \(\sigma_y:\sigma_x = 1:-1\), are shown in Fig. 14. The lamina are in a state of pure shear in the local (material) coordinate system. MCT is in good agreement with the experimental data to about 2% laminate strain where shear failure of the laminate is predicted. Using the generalized plane strain transformation equation [Eq. (4)], the local shear strain, \(\gamma_{12}\), corresponding to laminate strains \(\varepsilon_x = \varepsilon_y = 2\%\) and \(\gamma_{xy} = 0\), is 4% which is equal to the shear failure strain provided by the organizers. This result is expected since the MCT shear failure parameters were derived from this data. At failure the experimental lamina shear strain, \(\gamma_{12,\text{ex}}\), is calculated to be approximately 20% which is five times that provided by the organizers. Again, the constraining influence of the laminate stacking sequence (in this case a [0°/90°] laminate rotated 45°) is believed to result in enhanced stress–strain ultimate values that cannot be predicted by extrapolation of the UD shear data. MCT under-predicts laminate failure strength by 22%.

3. General discussion

We are pleased with our failure simulations of the four different composite materials and fourteen cases provided by the organizers of the failure exercise. The generally good analytical versus experimental correlation validates the approach of predicting composite damage based on failure of the constituents. We modified our original predictions in only three of the fourteen cases. In all cases, we simply failed to allow our nonlinear finite-element analysis to reach equilibrium and prematurely stopped the analysis.

The MCT approach provides a high resolution window on laminate behavior by highlighting initial and intermediate damage states. Initial failure is often difficult to delineate experimentally but is important in structural design because it reduces stiffness and produces nonlinear, inelastic laminate behavior. Constituent failure (typically the matrix) begins as scattered micro-ruptures that must multiply and coalesce to some threshold value before manifesting itself as a noticeable change in experimentally monitored metrics such as displacement, strain, or load. The predictive capability of micro/macro analysis techniques such as MCT can help the experimentalist discern subtle changes in the

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Fig. 13. Biaxial, \(\sigma_y:\sigma_x = 1:1\), stress–strain curves for a [±45°]₅ laminate made from E-glass/MY750/HY917/DY063.

Fig. 14. Biaxial, \(\sigma_y:\sigma_x = 1:-1\), stress–strain curves for a [±45°]₅ laminate made from E-glass/MY750/HY917/DY063.
metrics when testing for initial and intermediate damage states.

The experimental data, in particular cases 1, 2, and 9, indicate that shear strength may be enhanced in the presence of normal stress. Recognition of the magnitude of this phenomena is one of the lessons learned by the authors from this exercise. To account for increasing lamina strength, a normal-shear interaction term can be added to the failure criteria. The full constituent-based failure criteria proposed by the authors [3]

\[
K_{2m}I_m^2 + \frac{1}{2S_{23m}}I_{3m} + \frac{1}{S_{12m}}I_{4m} = 1
\]

includes a term

\[
\frac{1}{\left(\frac{S_{22m}}{S_{23m}} \pm \frac{S_{22m}}{S_{33m}} \right)^{2}} \left( 1 - \frac{S_{33m} + S_{22m}}{2S_{23m}} \right)
\]

that acts implicitly as an interaction term by taking on a negative value for materials that exhibit strong shear-normal interaction (stress invariant contributions are always positive definite). The term \(K_{2m}\) was omitted in the current research due to insufficient data to determine a value with confidence. In previous research [4], we saw a stronger interaction in an E-glass/vinylester (E-glass/8084) material \((K_{2m} = \text{negative in the presence of tension})\) than in the carbon/epoxy (AS4/3501) or boron/epoxy (boron/5505). \(\pm K_{2m}\) values for these materials are presented in Table 2. Incorporating the effects of \(K_{2m}\) in the present exercise would certainly be of interest.

As mentioned in our contribution to Part A [3] of the failure exercise, data from off-angle, balanced, symmetric laminates, \([\pm 0]_S\), provide an excellent basis for determining a best fit of failure parameters \(K_{12m}\), \(K_{23m}\), and \(K_{13m}\). Hence, some of the laminates analyzed as part of this exercise, e.g., \([\pm 55]_S\) and \([\pm 45]_S\), could be used as inputs to “tune” the failure analysis.

Additionally, the fifth transversely isotropic stress invariant,

\[
I_3 = \sigma_{22}^2 \sigma_{12}^2 + \sigma_{13}^2 \sigma_{13}^2 + 2\sigma_{12} \sigma_{13} \sigma_{23},
\]

which was not included in the quadratic failure criteria proposed, may provide an explicit term to account for normal-shear stress interaction. Modifying the MCT failure to include this term may be the subject of future research.

The experimental data, in particular cases 10, 12, and 14, indicate that lamina strengths may be enhanced by the constraining action of adjacent lamina, especially if they are oriented orthogonally. The most effective way to account for this effect may be to derive failure parameters from cross-ply rather than unidirectional laminate tests. In previous research [4], the lamina shear stress–strain behavior was taken from uniaxial compressive tests of \([\pm 45]_S\) laminates with good success. Likewise, lamina transverse strengths (and associated constituent strengths) could be taken from uniaxial tests of \([0°/90°]_S\) laminates. This method has been used to determine the compressive strength of carbon/epoxy lamina [9].

The reader is reminded that a comparison between the predictions of the present method and those of other methods is made in Ref. [10].

4. Concluding remarks

There is a severe lack of stress–strain and failure data on composite laminates subjected to multiaxial loads. The scarcity of data is due in large part to the difficulty and expense in generating it. There is also a bewildering variety of approaches to predicting failure of composite structural laminates. The need for a venue to objectively evaluate the composite community’s collective ability to predict failure of structural composites under in-service load conditions has been painfully missing. This exercise should prove to be a seminal work in this area and the organizers (Hinton, Kaddour and Soden) are to be commended for their hard work and valuable contribution to the body of engineering knowledge.

The MCT approach to conducting composite failure analysis was intended from the beginning to be used as a main stream design tool. The method was implemented into the finite element frame work, formulated to minimize user input, be computationally fast, completely three dimensional, and generally produce conservative failure values. Constituent-based failure criteria form the basis for a progressive failure analysis allowing for accurate composite laminate structural analysis in pre-, ongoing and post- damage conditions. Because information is developed on a level at which failure initiates, load redistribution to other parts of the structure as well as the remaining constituents can be efficiently included. This load redistribution allows a designer to track failure as it occurs, region by region, and reduce the stiffness and strength of damaged areas without necessarily declaring total structural failure. Changes in structural design, to remedy weaknesses, can be concentrated to specific areas leading to a more optimized design.

We view capturing improved failure parameters for the MCT analysis as the best way to improve this design tool. We have made multiple comments throughout the
paper on how to improve upon these failure strength estimates. However, we believe the fundamental MCT methodology represents a simplistic, elegant, and accurate approach to failure analysis of composites.

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References