A Multicontinuum Theory for Thermal-Elastic Finite Element Analysis of Composite Materials

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ABSTRACT: Structural analysis of composite materials is severely limited by the lack of constituent information. This information is generally unavailable due to the continuum hypothesis associated with the geometric scale of the problem. Micromechanics has been the traditional approach to achieving insights into constituent behavior in a composite material. However, modelling micromechanical details in a structural application is frequently impractical.

In this paper, by example of linear elasticity, we suggest a viable bridge between structural analysis and micromechanics. The approach utilizes detailed finite element models of a representative volume element as a one-time preprocessor to characterize the structural stiffness of the material. This information is used to implement an analytical expression for constituent stress and strain fields (in the continuum sense) as a function of the structural fields in a conventional finite element analysis. The computational premium is minimal while the benefits and additional insights regarding constituent behavior are significant. Numerical results for constituent as well as structural fields are presented for thermal-elastic behavior of a composite plate with a hole. The results indicate dramatic differences in the constituent stress fields for a comparable structural stress caused by thermal and mechanical loads, respectively. A brief discussion of the potential extension to inelastic analysis is also included.

INTRODUCTION

THE SUCCESS OF modern continuum mechanics in modelling composite material behavior is truly remarkable. For instance, the general theories of elasticity, plasticity, and viscoelasticity all rely on the continuum hypothesis. However, while continuum mechanics has provided a powerful means of studying the physics of deformation of composite materials, there are situations when the

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continuum hypothesis is simply inadequate. These problems are generally associated with inelastic behavior of composites and are mainly attributed to the necessity to homogenize two distinctly different materials into a single continuum.

High temperature environments profoundly amplify all the problems associated with a mechanical analysis of composite structures. For instance, when a composite structure is heated, severe internal stresses develop at the constituent level caused by mismatched thermal expansions of the constituents. A classic single continuum structural analysis is unable to detect these stresses as the thermal expansion coefficients of the composite are homogenized into a single value. Hence, current analysis capabilities are extremely limited in thermo-mechanical applications.

One approach to structural analysis of composites is to treat the material as a multiphase continuum (multicontinuum), thereby allowing one to retain information at the constituent level. Consistent with a single continuum theory, each constituent is assumed present at every point in the continuum. The fundamental difference is that the constituents retain their identity rather than being homogenized in some sense. The difficulty with this approach is to relate the constituent behavior to the global response. Specifically, a mathematical means of accounting for the interactions between constituents must by developed.

In what follows, we develop a set of kinematic constraints relating the constituent deformation fields for thermal-elastic behavior. The constraints allow one to determine constituent stress and strain fields in the course of a structural analysis. The basic form of the constraints is seen to be generally valid for inelastic material behavior also. The development follows some fundamental relationships between constituent properties and the properties of the composite which first appeared in Hill [1]. Implicit to the development are the concepts of a representative volume element (RVE), also discussed by Hill [1], and macroscopically uniform stresses and strains associated with the RVE of a statistically homogeneous medium. Detailed discussions of these concepts are to be found in Hashin [2] and Nemat-Nasser [3].

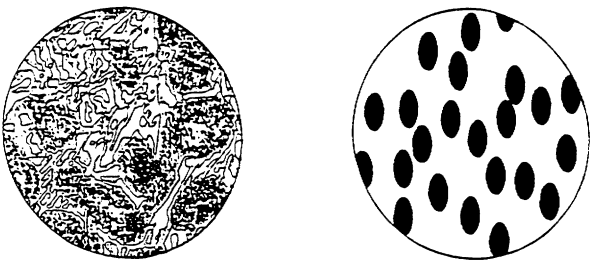
Results for the classic plate with circular hole problem are presented for both thermal and mechanical loads. Finally, we include a discussion of the apparent advantages the present theory offers, including the anticipated benefits of extending the theory to inelastic analyses.

Both direct tensor notation and contracted matrix notation are employed. In direct notation, vectors are denoted by boldface characters while second order tensors are underscored with a tilde. Fourth order tensors are both bolded and underscored with a tilde.

THE MULTICONTINUUM CONCEPT AND THE ROLE OF MICROMECHANICS

To fully appreciate the idea of a multicontinuum, one must first develop a physical interpretation for a continuum point as depicted in Figure 1(a). Loosely defined:

A continuum point is a geometric point in space conceived as having no



(a) (b)

Figure 1. *(a) Microscopic view of a continuum point. (b) Microscopic view of a continuum point with two clearly identifiable constituents.*

volume but that retains properties associated with a finite volume by taking mathematical limits.

Thus, properties at a point in the continuum are based on a volume average of the properties "within" the point. In effect, the volume averaging represents an averaging of microscopic material variables, thus rendering the macroscopic properties. For instance, the continuum definition of the stress tensor at a point is given by:

$$\underset{\sim}{\sigma} = \frac{1}{V_r} \int_R \underset{\sim}{\sigma}(\mathbf{x})\, dV \tag{1}$$

where the domain R represents the continuum point and V is the volume associated with the point in the averaging process.

Now consider a composite material consisting of two clearly identifiable constituents, i.e., a reinforcing material embedded in a matrix material. As illustrated in Figure 1(b), one can clearly identify both constituents within a continuum point. Now extend the previously discussed averaging notion to the constituent level. For example, a constituent stress tensor for the matrix and reinforcement may be defined as follows:

$$\underset{\sim}{\sigma}_r = \frac{1}{V_r} \int_{R_r} \underset{\sim}{\sigma}(\mathbf{x})\, dV \tag{2}$$

and

$$\underset{\sim}{\sigma}_m = \frac{1}{V_m} \int_{R_m} \underset{\sim}{\sigma}(\mathbf{x})\, dV \tag{3}$$

where $R = R_m \cup R_r$. The subscripts r and m denote the reinforcement and matrix respectively. The total (composite) stress of Equation (1) is readily recovered from the constituent stresses of Equations (2) and (3) by noting:

$$\underset{\sim}{\sigma} = \phi_r \underset{\sim}{\sigma}_r + \phi_m \underset{\sim}{\sigma}_m \tag{4}$$

where ϕ_r and ϕ_m denote the volume fractions of the reinforcement and the matrix respectively.

It is often convenient to work with the partial stress of each constituent given by

$$\underset{\sim}{t}_r = \phi_r \underset{\sim}{\sigma}_r \tag{5}$$

and

$$\underset{\sim}{t}_m = \phi_m \underset{\sim}{\sigma}_m \tag{6}$$

Hence, the partial stress is simply the true constituent stress weighted by the constituent volume fraction. Comparing Equations (4–6) gives

$$\underset{\sim}{\sigma} = \underset{\sim}{t}_r + \underset{\sim}{t}_m \tag{7}$$

At this point we introduce the role of micromechanics as it provides a versatile means of implementing the averaging process. Consider the continuum point of Figure 1(b). To begin a micromechanics analysis we first assume a periodic unit cell geometry as representative of the microstructure. For instance, for a unidirectional fibrous composite, hexagonal or square packing of the fibers is commonly assumed. For particulate reinforced composites, body-centered cubic and face-centered cubic arrangements have been studied. Perhaps the best unit cell is a random packing in which enough fibers or particles are included to yield a statistically accurate form of the macroscopic continuum. The influence of assumed microstructural geometry on the constitutive response of composites has been studied elsewhere. For instance, Brockenbrough et al. [4] have studied the linear and nonlinear behavior of metal matrix composites using various unit cells. It was generally found that linear elastic behavior is a relatively weak function of reinforcement distribution while nonlinear behavior can be a strong function of the assumed distribution. The validity of computational micromechanics in characterizing composite material behavior is discussed by Rammerstorfer et al. [5]. The limitation of the uniform field/RVE model presupposed in most micromechanical models is analyzed by Ye [6].

For discussion purposes, consider a unidirectional fibrous composite with hexagonal packing. A microstructure of this type and an associated unit cell is shown in Figure 2. In the context of a continuum field, the unit cell represents a fraction of a continuum point. Yet because of the assumed periodicity, the unit cell is mathematically representative of the entire point. Therefore, the properties associated with a continuum point in a structural analysis may be determined by volume averaging the properties within the unit cell.

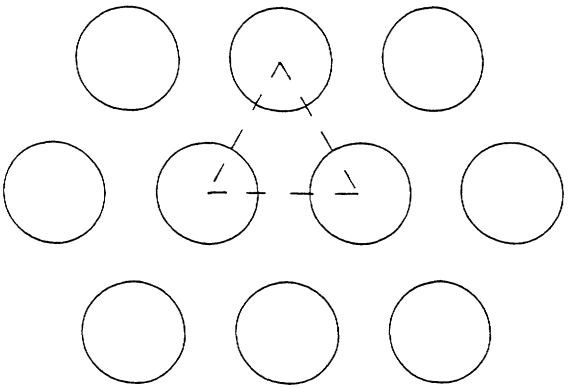


Figure 2. *Microstructure and periodic unit cell for a hexagonal packing.*

Equation (1) represents an averaging process that may be extended to the constituent level using Equations (2) and (3) as applied to the unit cell. Hence, the volume averaged stresses defined by Equations (2) and (3) represent constituent stresses at a point whose composite (structural) stress is defined by Equation (1). Using finite element micromechanics, it is straightforward to evaluate Equations (1–3) by numerical integration.

Consistent with the previous discussion on stresses, the strain field at a point in a structural analysis is given by (Laws [7])

$$\underset{\sim}{\epsilon} = \frac{1}{V}\int_R \underset{\sim}{\epsilon}(\mathbf{x})\, dV \tag{8}$$

where the integral is carried out over the volume of the unit cell. We also introduce constituent strain fields for a multicontinuum analysis given by

$$\underset{\sim}{\epsilon}_r = \frac{1}{V_r}\int_{R_r} \underset{\sim}{\epsilon}(\mathbf{x})\, dV \tag{9}$$

$$\underset{\sim}{\epsilon}_m = \frac{1}{V_m}\int_{R_m} \underset{\sim}{\epsilon}(\mathbf{x})\, dV \tag{10}$$

where

$$\underset{\sim}{\epsilon} = \phi_r \underset{\sim}{\epsilon}_r + \phi_m \underset{\sim}{\epsilon}_m \tag{11}$$

In summary, the physical significance of a continuum stress or strain field in

the context of a unit cell analysis should be clear. In particular, the continuum stress or strain for a structure may be obtained by volume averaging the appropriate field for the unit cell. Therefore, the multicontinuum concept simply represents an extension of the single-continuum theory by clearly identifying continuum properties for each constituent.

A MULTICONTINUUM THEORY

Here we transition from micromechanics and point stress definitions to a multicontinuum structural analysis. That is, the volume averaged stresses and strains at a point become spatially varying tensor fields within a structure. Relationships governing constituent fields as a function of the composite fields were first developed by Hill [1]. Here we generalize the development to include thermal effects. Various forms of the thermo-elastic result can also be found elsewhere, e.g., Aboudi [8].

Consider a two-phase composite consisting of a reinforcing material embedded in a matrix material. In contracted matrix notation the constitutive law for the composite may be written as

$$\{\sigma\} = [C](\{\epsilon\} - \{\epsilon_o\}) \tag{12}$$

where $[C]$ is the composite stiffness matrix and $\{\epsilon_o\}$ is a vector of initial strains due to thermal expansion. Similarly, at the constituent level we can write

$$\{\sigma_r\} = [C_r](\{\epsilon_r\} - \{\epsilon_{ro}\}) \tag{13}$$

and

$$\{\sigma_m\} = [C_m](\{\epsilon_m\} - \{\epsilon_{mo}\}) \tag{14}$$

In the above, the stiffness matrices $[C_r]$ and $[C_m]$ are known from the constituent material properties. Using Equation (4) the structural stress may be represented as

$$\{\sigma\} = \phi_r\{\sigma_r\} + \phi_m\{\sigma_m\} \tag{15}$$

Substituting Equations (12–14) into Equation (15) and noting Equation (11) gives

$$\phi_r[C_r](\{\epsilon_r\} - \{\epsilon_{ro}\}) + \phi_m[C_m](\{\epsilon_m\} - \{\epsilon_{mo}\}) = [C](\phi_r\{\epsilon_r\} + \phi_m\{\epsilon_m\} - \{\epsilon_o\}) \tag{16}$$

Solving for $\{\epsilon_r\}$ gives

$$\{\epsilon_r\} = -\frac{\phi_m}{\phi_r}([C] - [C_r])^{-1}([C] - [C_m])\{\epsilon_m\} + \frac{\Theta}{\phi_r}([C] - [C_r])^{-1}([C]\{\eta\} - \phi_r[C_r]\{\eta_r\} - \phi_m[C_m]\{\eta_m\}) \tag{17}$$

In the above, Θ is the temperature relative to some reference temperature. The vectors $\{\eta\}$, $\{\eta_r\}$, and $\{\eta_m\}$ contain thermal expansion coefficients for the composite, the reinforcement, and the matrix, respectively. Thus, the following substitutions were made:

$$\Theta\{\eta\} = \{\epsilon_o\}, \quad \Theta\{\eta_r\} = \{\epsilon_{ro}\}, \quad \Theta\{\eta_m\} = \{\epsilon_{mo}\} \tag{18}$$

Alternatively, we can rewrite Equation (17) as

$$\{\epsilon_r\} = [A]\{\epsilon_m\} + \frac{\Theta}{\phi_r}\{a\} \tag{19}$$

where

$$[A] = -\frac{\phi_m}{\phi_r}([C] - [C_r])^{-1}([C] - [C_m])$$

and

$$\{a\} = ([C] - [C_r])^{-1}([C]\{\eta\} - \phi_r[C_r]\{\eta_r\} - \phi_m[C_m]\{\eta_m\}) \tag{21}$$

In direct tensor notation, the constraint equation appears as

$$\underset{\sim}{\epsilon}_r = \underset{\sim}{\mathbf{A}} : \underset{\sim}{\epsilon}_m + \frac{\Theta}{\phi_r}\underset{\sim}{a} \tag{22}$$

In the above, $\underset{\sim}{\mathbf{A}}$ is a fourth order tensor coupling mechanical effects while $\underset{\sim}{a}$ is a second order tensor that accounts for thermal strain effects. Specific values for these tensors are clearly dependent on the microstructure as well as the constituent material properties. In particular, noting Equations (20) and (21); $[C_r]$, $[C_m]$, $\{\eta_r\}$, and $\{\eta_m\}$ are assumed known while $[C]$ and $\{\eta\}$ are extracted from micromechanics analyses.

Now consider how one might invoke the constraint of Equation (19) in a finite element environment. One approach is to simultaneously model both constituent displacement fields and enforce the relationship via penalty constraints (Hansen and Garnich [9]). Though conceptually straight forward and theoretically well founded, this approach does not readily produce satisfactory results. The problem is rooted in the fact that the constraint relates the strains which are functions of the first derivatives of displacements. In the context of finite elements, in order

to satisfy such a constraint everywhere we require continuous first partial derivatives in displacements or C^1 elements. Unfortunately, C^1 elements are not an attractive alternative to the widely used C^0 elements which produce continuous strain fields only for trivial problems (e.g., constant stress).

An alternative approach is to abandon the calculation of constituent displacements and simply calculate constituent strains and stresses from the composite solution. This is accomplished by substituting Equation (19) into Equation (11) and solving for $\{\epsilon_m\}$ to yield

$$\{\epsilon_m\} = (\phi_m[I] + \phi_r[A])^{-1}(\{\epsilon\} - \Theta\{a\}) \tag{23}$$

where $[I]$ is the identity tensor. Then, again using Equation (11) we can calculate

$$\{\epsilon_r\} = \frac{1}{\phi_r}(\{\epsilon\} - \phi_m\{\epsilon_m\}) \tag{24}$$

These last two equations allow the calculation of constituent strains at any point in the structure. The constituent stresses may then be determined using Equations (13) and (14). As stated previously, the isothermal equivalent of Equation (23) was developed by Hill [1]. However, the motivation for doing so was entirely different. Hill's motivation was to estimate the stiffness matrix for a composite material. Hill proposed that by knowing a constituent strain field, one could predict the modulus of the composite. Here, we take the opposite view, i.e., by calculating the composite stiffness from finite element micromechanics, we are able to determine the constituent stress and strain fields everywhere in the structure.

The above solution for constituent variables is "closed form." That is, beyond the assumption of a microstructural geometry, there are no approximations beyond those of standard linear elastic analysis.

DEMONSTRATION MODEL

The fundamental feature which separates the multicontinuum theory from a single continuum approach is the ability to generate constituent stress and strain fields throughout the body in the course of a structural analysis. Here we seek to show the value of such information in a structural analysis.

Consider a continuous fiber boron/aluminum composite with the fibers running parallel to the X_1 direction. Hexagonal packing of the fibers is assumed with a 45 percent fiber volume. The hexagonal microstructure and the associated triangular unit cell geometry are depicted in Figure 2. The unit cell defines the geometry of the micromechanices model. The finite element mesh (Figure 3) contained 483 8 node isoparametric brick elements with $2 \times 2 \times 2$ integration. The boundary conditions for such a micromechanics model are nontrivial and have been previously described by Brockenbrough et al. [4]. The assumed isotropic material properites for the boron and aluminum are given in Table 1 and were taken from Dvorak et al. [10] and Gibson [11], respectively.

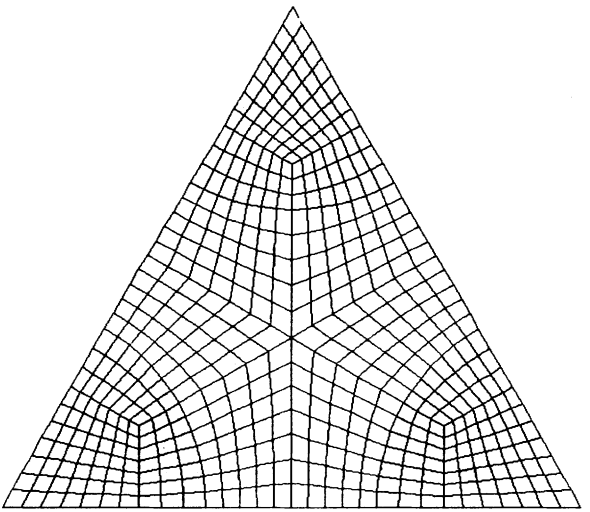


Figure 3. *Finite element mesh for the hexagonal packed periodic unit cell.*

To begin, the microstructure tensors, $\underset{\sim}{\mathbf{A}}$ and $\underset{\sim}{a}$, must be determined which require knowledge of the composite stiffness matrix $[C]$ and the thermal expansion properties of the composite. The composite stiffness matrix may be expressed in terms of 5 independent elastic constants for a transversely isotropic material, e.g., E_{11}, E_{22}, ν_{12}, ν_{23}, and G_{12}. These constants may be determined from a set of three finite element micromechanics load cases for a unit cell of the composite; specifically, longitudinal tension, transverse tension, and longitudinal shear. Thermal properties of the composite may be determined from a single thermal expansion test in which the unit cell is subjected to a uniform change in temperature. The material coefficients determined from these load cases are given in Table 2.

The determination of the microstructure tensors, $\underset{\sim}{\mathbf{A}}$ and $\underset{\sim}{a}$, from a micromechanics analysis represents a vital link between micromechanics and structural analysis. Clearly such a connection between the micro- and macro-response is necessary if one truly hopes to capture the response of the structure. Further-

Table 1. Material properties for boron and aluminum.

	Boron	Aluminum
Young's Modulus (GPa)	400.0	72.39
Shear Modulus (GPa)	166.7	27.22
Thermal expansion coefficients (°C^{-1})	16.2(10)$^{-6}$	23.4(10)$^{-6}$

Table 2. Elastic constants for a unidirectional boron/aluminum composite with 45 percent fiber volume and hexagon packing. Fibers are oriented along the X_1 axis.

E_{11} = 220.1 GPa	G_{12} = 53.27 GPa
$E_{22} = E_{33}$ = 137.6 GPa	η_{11} = 17.63(10)$^{-6}$°C^{-1}
ν_{12} = 0.2651	$\eta_{22} = \eta_{33}$ = 20.61(10)$^{-6}$°C^{-1}
ν_{23} = 0.3582	

more, it should be emphasized that once the microstructure tensors are determined, micromechanics is no longer needed.

To demonstrate the value of constituent information, consider a transverse tension test of the boron/aluminum composite in which σ_{22} is nonzero. Specific results for the constituent stress fields are given in Table 3. For comparison purposes, the volume averaged stress fields obtained from a finite element micromechanics analysis are provided along with stresses given by the closed form work previously developed. The results are numerically exact for all stress components.

The first row in the micromechanics portion of Table 3 represents the composite (structural) stress applied to the body while rows two and three represent the *partial stresses* of the fiber and matrix, respectively. It is significant to note that for each stress component, the sum of the partial stresses equals the structural stress. This result is a numerical statement of Equation (7) and has been proven analytically in Laws [7].

Consistent with the micromechancis solution, the multicontinuum theory predicts a *structural stress* of zero for the two off-axis directions. However, examination of the constituent results shows significant off-axis stresses. In fact, the

Table 3. Stress comparisons between a micromechanics analysis and the multiphase continuum theory for a transverse tension loading of 1000 kPa.

	Micromechanics					
	σ_{11}	σ_{22}	σ_{33}	σ_{12}	σ_{13}	σ_{23}
Composite	0	1000	0	0	0	0
Fiber	−116	528	−25.1	0	0	0
Matrix	116	472	25.1	0	0	0
	Multiphase Continuum Theory					
	σ_{11}	σ_{22}	σ_{33}	σ_{12}	σ_{13}	σ_{23}
Composite	0	1000	0	0	0	0
Fiber	−116	528	−25.1	0	0	0
Matrix	116	472	25.1	0	0	0

matrix stress t_{11m} is approximately 25 percent of the matrix stress in the loaded direction. This information is simply unavailable in the course of a conventional structural analysis. Furthermore, it is precisely this type of information which can lead to improved analytical predictions of phenomena such as matrix cracking and constituent debonding.

As a second demonstration of the value of constituent information, consider a plate with a hole composed of the boron/aluminum unidirectional composite shown in Figure 4. A standard finite element formulation, utilizing 4 node bilinear plane stress elements was employed. Results from two uniquely different load cases for the plate are presented. The first is a stress field caused by non-uniform temperature field. The temperature field is a static one obtained by giving the edge of the hole a fixed temperature of 5.56°C (10°F) above the reference temperature and placing convective boundary conditions on the exterior. Figure 5 shows the finite element mesh and stress contours generated by the thermal loading. The peak stress state at the hole is nearly one-dimensional with a stress of $\sigma_{11} = -8056$ kPa.

As a second load case, consider loading the edge of the plate with a mechanical load as shown in Figure 4. The magnitude of the edge loading was chosen so as to match the peak composite stress, σ_{11}, at the hole caused by the thermal loading. Stress contours for the mechanical loading are shown in Figure 6. To give credibility to the finite element model, the maximum stress at the edge of the hole was compared to an analytical solution given by Lekhnitskii [12] for an infinite orthotropic plate with circular hole. The finite element result for the stress con-

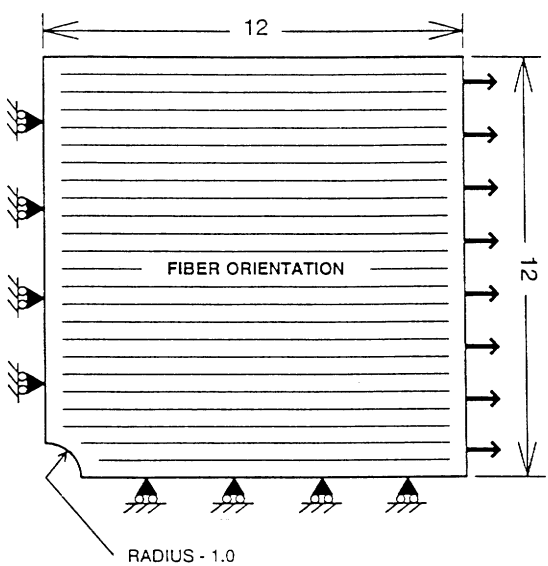
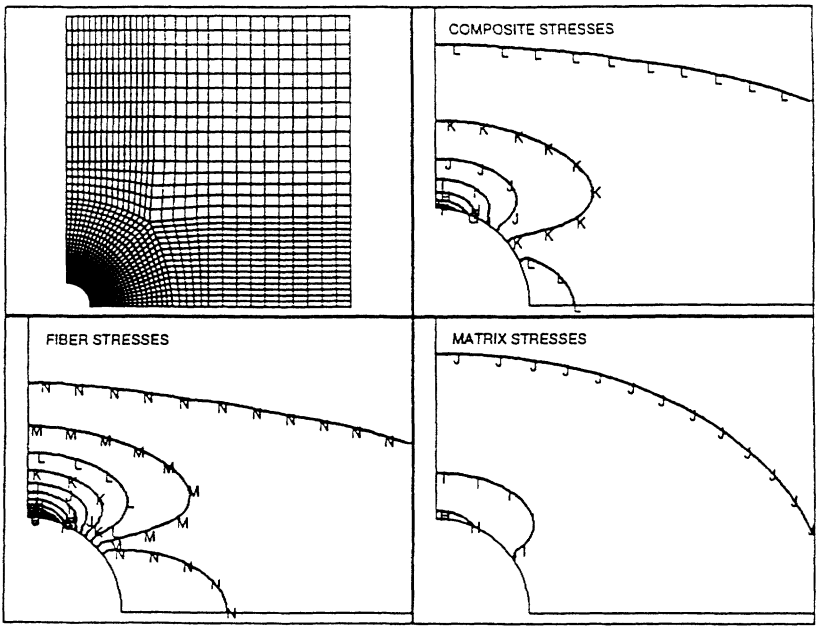


Figure 4. *Composite plate with a hole showing fiber orientation.*



Stress Contours (kPa)

A = -12,000	F = -7200
B = -11,400	G = -6200
C = -10,300	H = -5200
D = -9300	I = -4100
E = -8300	J = -3100

Figure 5. *Stress contours in the composite, fibers, and matrix for a composite plate subjected to a nonuniform temperature field.*

centration factor in the present finite plate is 3.57 while the analytical solution for an infinite plate is 3.61.

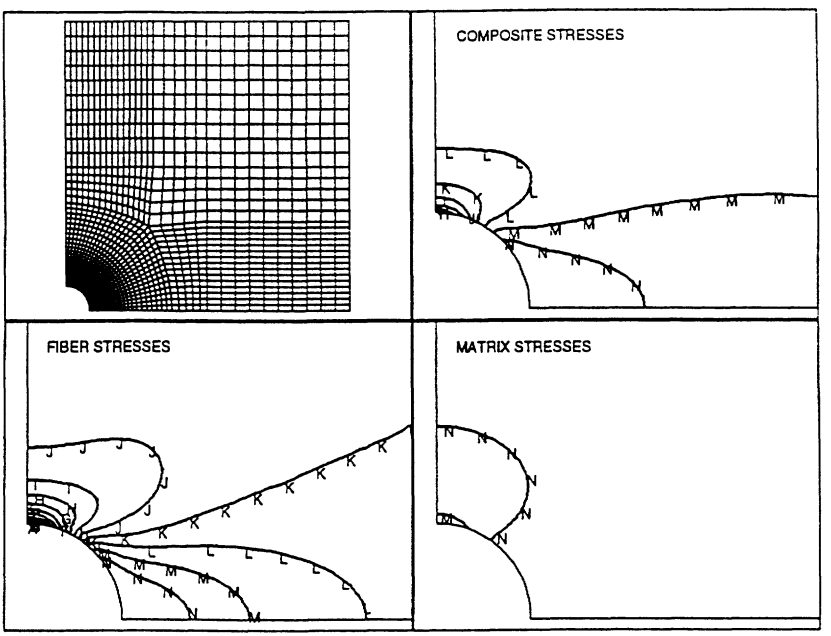
Table 4 shows the constituent partial and structural stresses for the σ_{11} component generated by these two load cases. The table shows the value of constituent information in dramatic fashion. Even though the peak stresses are identical for the composite, the constituent stresses are radically different. In particular, the matrix stresses are different by a factor of 2 for the two load cases. Clearly, the response of the plate to these loads will be significantly different. This illustrates a benefit of the multicontinuum theory which truly distinguishes it from a conventional structural analysis.

To the knowledge of the authors, this is the first time stress plots have been pre-

sented in the open literature for the constituent materials within a composite as a routine product of a general finite element structural analysis. This information adds significant insight into the response of a structure with a minimal computational premium. Specifically, the multicontinuum theory required four micromechanics solutions and an additional constitutive computation for strains as defined by Equations (23) and (24) prior to the normal constitutive computation for stresses for each integration point. Further, once the micromechanics solutions have been obtained for a particular composite, they need never be computed again for that material.

EXTENSIONS TO INELASTIC BEHAVIOR

A multicontinuum theory represents a premier analysis capability in that it



Stress Contours (kPa)

A = −14,500	F = −9300
B = −13,500	G = −8300
C = −12,400	H = −7200
D = −11,400	I = −6200
E = −10,300	J = −5200

Figure 6. *Stress contours in the composite, fibers, and matrix for a composite plate subjected to a uniform edge load.*

Table 4. Peak stress comparisons of the thermal and mechanical loadings of a plate with a hole.

Stress (kPa)	Mechanical σ_{11}	Thermal σ_{11}
Composite	−8056	−8056
Fiber (partial stress)	−6525	−4842
Matrix (partial stress)	−1531	−3214

allows one to perform structural analysis while retaining information regarding the constituent stress and strain fields. This capability offers unique possiblities when studying the inelastic behavior of a structural system. For instance, plastic and viscoelastic behaviors are common in many matrix materials used for composites. A multicontinuum theory allows one to drive the composite response using information at the constituent level.

As an example, consider subjecting a unidirectional graphite/epoxy composite to free thermal expansion caused by a uniform increase in a temperature field. After initially expanding, composites of this nature have been observed to shrink in the fiber direction at constant temperatures. This phenomenon has been referred to as reverse creep and may be attributed to stress relaxation in the viscoelastic matrix material. However, the composite is stress free at the structural level and hence no stress relaxation may be predicted or observed in a conventional finite element analysis at the structural level. In contrast, the multicontinuum theory is capable of tracking this relaxation as constituent stress information is a routine part of the structural calculations. This illustrates the power of driving the response of the composite at the constituent level.

A particularly important problem in structural analysis is the ability to quantify microstructural damage which may occur at the constituent level. A multicontinuum formulation provides a basis for modelling failure or damage at the constituent level while performing a structural analysis. A related problem in composite structural applications is the potential for bond failure at a constituent interface. Analysis of bond failures at the structural level transcends the capability of a single continuum theory. In contrast, a multicontinuum theory is ideally suited for modelling such failures because the coupling between constituents appears explicitly in the analysis and is a direct measure of bond strength.

DISCUSSION

In closing, it is essential to place the multicontinuum theory in the context of the major structural analysis techniques being studied today. Due to the simplicity and elegance of continuum mechanics, the homogenized single continuum approach remains the most prominent analysis technique for composite structures. This approach has led to significant advances in anisotropic plasticity, viscoelasticity, and viscoplasticity. However, a single continuum theory is unsuitable for modelling many of the deformation mechanisms associated with the inelastic be-

havior of composites. Particularly weak areas include damage mechanics and high temperature behavior. The chief difficulty in each case is the lack of ability to isolate the material response at the constituent level.

There are other approaches to composite structural analysis that bring constituent information into the solution process. Perhaps the most prominent are due to Aboudi [8,13] with the method of cells, and Dvorak [14,15] with transformation theory. Both methods are relatively mature, having been applied to various inelastic material models. However, in contrast to the approach suggested here, these models are coupled to the structural analysis in the solution process and therefore bring a high computational premium. Simplifying these micromechanical models can lessen the computation but at the cost of reduced accuracy.

The approach outlined here does not directly couple the micromechanics model to the structural analysis. Rather, analytical expressions providing micromechanical information are obtained as a precursor to the general structural analysis. As such, there is no motivation to simplify the micromechanics model for the sake of economy. Therefore, we are free to use micromechanical models of high degrees of complexity, e.g., random unit cells. This versatility is important in light of the conclusions drawn by Brockenbrough [4] where it was found that nonlinear behavior is strongly influenced by the assumed reinforcement distribution.

Consider again the manner in which micromechanical information is introduced in the multicontinuum theory presented here-in. In particular, micromechancial effects are introduced into the theory through the strain coupling constraint given by Equation (19). The primary obstacle in this approach is developing correct expressions for the composite stiffness matrix $[C]$ for nonlinear behavior. However, once $[C]$ has been defined for a nonlinear behavior, it is valid for all points in the continuum.

In summary, the success of modern continuum mechanics in modelling the elastic behavior of composites is undeniable. The multicontinuum theory simply represents a logical extension of the classical continuum approach by allowing each constituent to retain its identity. Hence, the power and conciseness of a continuum approach is retained while at the same time, a great number of analysis barriers associated with homogenization of constituents are eliminated.

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