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Progressive failure modeling of woven fabric composite materials using multicontinuum theory

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Abstract

Failure of composite materials often results from damage accumulation in the individual constituents (fiber and matrix) of the composite. At times, damage may even be limited to a single constituent. The ability to accurately predict not only ultimate strength values but also intermediate constituent level failures is crucial to the success of introducing composite materials into demanding structural applications.

In this paper, we develop two progressive failure models for the analysis of a plain weave composite material. The fo.rmulations are based on treating the weave as consisting of separate but linked continua representing the warp fiber bundles, fill fiber bundles, and pure matrix pockets. Retaining constituent identities allows one to access constituent (phase averaged) stress fields that are used in conjunction with both a stress based and damage based failure criterion to construct a nonlinear progressive failure algorithm for the woven fabric composite material. The MCT decomposition and the nonlinear progressive failure algorithm are incorporated within the framework of a traditional finite element analysis.

The constituent based progressive failure algorithm combined with both the stress based and damage based failure criteria are compared against experimental data for a plain weave, woven fabric composite under various loading conditions. The analytical results from the damage based approach show a marked improvement over the stress based predictions and are in excellent agreement with the experimental data.
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1. Introduction

The continually expanding use of composite materials in large structural applications places a premium on understanding their survivability. An essential aspect of modeling survivability is the ability to quantify damage and its effect on structural performance prior to ultimate failure. Damage in a composite often manifests itself in the form of submicrocrack accumulation occurring in the matrix constituent. An example of intermediate damage in a plain weave glass–fabric composite is shown in Fig. 1. The milky white areas are a clear indicator of matrix damage occurring in the fiber bundles running transverse to the loading direction.

The situation where damage initiates in a constituent of a composite prior to catastrophic failure is common. From a modeling perspective, it is extremely desirable to quantify the extent of constituent damage and its effect on structural performance in the course of a traditional structural analysis. By traditional analysis, we mean a standard nonlinear structural code capable of modeling progressive damage without having to rely on micromechanical models during the course of an analysis. The macro–micro approach is

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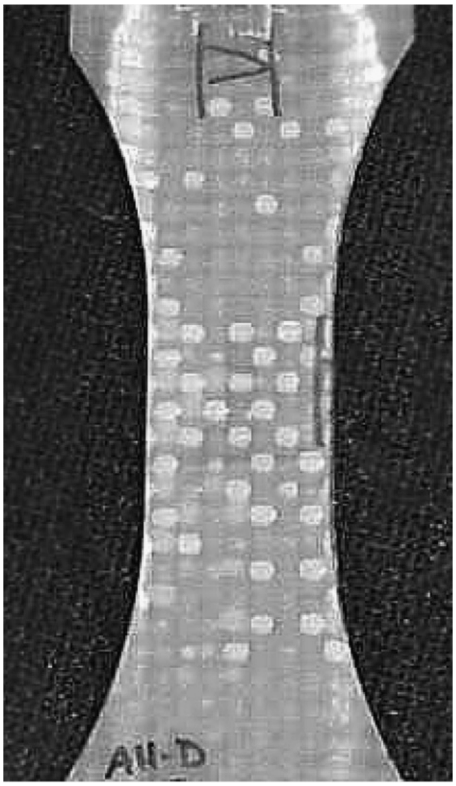

Fig. 1. Photograph of woven fabric uniaxial tension specimen.

not only undesirable from the standpoint of the amount of information generated; it is also numerically untenable for any meaningful large scale structural analysis. Thus the analyst is confronted with the paradox of seeking constituent level information to model damage in a composite while retaining the fundamental analysis at the structural level.

Multicontinuum theory (MCT) represents an attractive compromise between standard large scale structural analyses utilizing homogenized stress/strain fields and the detailed macro/micro modeling approach. MCT relies on the basic premise that a ‘continuum point’ for a composite is composed of separate but linked continua comprised of the individual constituents. Retaining the constituents’ identities allows one to access continuum (phase averaged) constituent stress and strain fields. Knowledge of constituent stresses and strains allows one to implement continuum damage models at the constituent level, thereby providing a vehicle to simulate physically observed damage phenomena.

In this work, we utilize a previously developed three constituent MCT decomposition algorithm in conjunction with newly developed constituent level failure criteria for plain weave fabric composites. An MCT analysis of woven fabrics involves decomposing the microstructure of Fig. 2 into three constituents consisting of warp fiber bundles, fill fiber bundles, and pure matrix pockets. Continuum stress/strain fields are generated for the constituents at every point in the structural analysis. In addition, continuum matrix stress and strain fields within the warp and fill bundles are accessed through a second MCT decomposition.

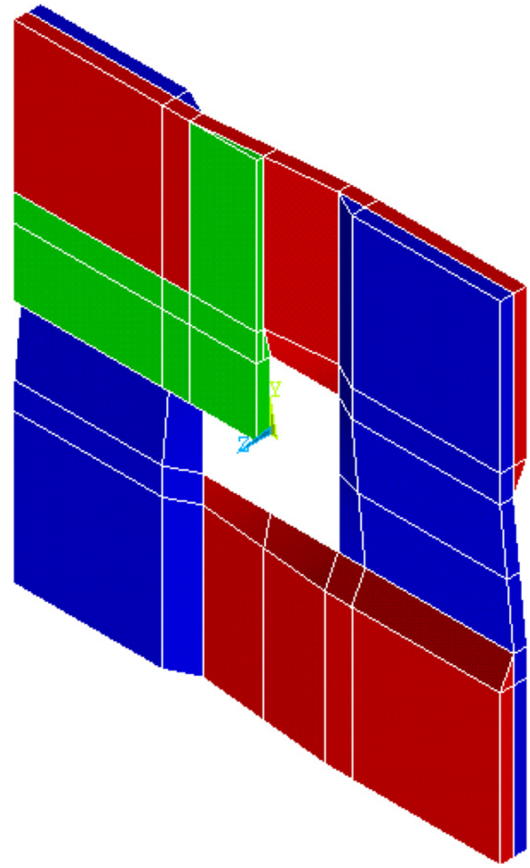

Fig. 2. Open, plain weave microstructure.

Two fundamentally different approaches to modeling intermediate failure within a woven fabric composite material are examined. The first approach examines constituent stresses of the fiber bundles and develops stress-based failure criteria for the fiber bundles. Both longitudinal (fiber) and transverse (matrix) failure modes at the bundle level are allowed. When failure is predicted in a fiber bundle, mechanical properties are adjusted for the bundle and the composite based on results from micromechanics. This binary approach to constituent failure has been used with success in the analysis of continuous fiber unidirectional composite laminates [1–3]. Again, we emphasize the damaged composite properties may be computed independent of any structural analysis and are known a priori to any analysis.

A second approach to modeling progressive failure in a woven fabric composite involves continuously degrading the matrix properties within the fiber bundles based on the amount of matrix damage resulting from submicrocrack accumulation in the material. The damage model is then related to material property degradation in the matrix, fiber bundles, and the composite. The resulting damage analysis produces a smooth macroscopic response for the composite that reflects observed inelastic material behavior.

In what follows, we provide a comparison of both failure models versus experimental data for various loading conditions of a plain weave composite material system. The results show that although the instantaneous degradation model works well under axial loading conditions, the continuous degradation model is superior and shows good agreement with both axial and shear loading cases. This increased correspondence is largely due to the inherent fact that matrix failure within composite materials is not an instantaneous phenomenon, but rather is generally caused by an accumulation of damage over a large loading range.

2. Overview of multicontinuum theory

To date, the primary application of the MCT decomposition algorithm in progressive failure analyses of composite materials has been limited to the two constituent case involving continuous fiber unidirectional composite materials. For the case of a two-phase composite consisting of constituents α and β there exists well known algebraic relations to decompose the composite stress/strain fields down to the constituent level. The decomposition first appeared in Hill [4] who developed the relations in an effort to estimate composite material stiffness properties. In the case of MCT, the relations are the same but the motivation is entirely the opposite. That is, we utilize known composite properties in conjunction with the decomposition of Hill to determine constituent stress/strain fields. We have relied on detailed finite element micromechanics models to compute the composite material mechanical properties.

We begin by noting the average (homogenized) value used to characterize the stress tensor at a 'continuum point' is derived by taking a volume average of all stresses in the region

$$\underset{\sim}{\sigma} = \frac{1}{V}\int_D \underset{\sim}{\sigma}(\mathbf{x})\mathrm{d}V, \tag{1}$$

where D is the region representing the continuum point.

In the case of a multicontinuum, we simply extend the definition of the continuum stress in Eq. (1) down to the constituent level. In particular, for a continuum point representing a two-phase composite, volume averaged stresses for constituents α and β may be expressed as

$$\underset{\sim}{\sigma}_\alpha = \frac{1}{V_\alpha}\int_{D_\alpha} \underset{\sim}{\sigma}(\mathbf{x})\mathrm{d}V, \tag{2}$$

and

$$\underset{\sim}{\sigma}_\beta = \frac{1}{V_\beta}\int_{D_\beta} \underset{\sim}{\sigma}(\mathbf{x})\mathrm{d}V, \tag{3}$$

where

$$D = D_\alpha \cup D_\beta.$$

The composite and constituent stress fields defined by Eqs. (1)–(3) lead directly to

$$\underset{\sim}{\sigma} = \phi_\alpha \underset{\sim}{\sigma}_\alpha + \phi_\beta \underset{\sim}{\sigma}_\beta, \tag{4}$$

where ϕ_α and ϕ_β are the volume fractions of constituents α and β, respectively.

Likewise, for strains we have:

$$\underset{\sim}{\varepsilon} = \phi_\alpha \underset{\sim}{\varepsilon}_\alpha + \phi_\beta \underset{\sim}{\varepsilon}_\beta. \tag{5}$$

Constitutive relations are required for the composite as well as the constituents. Assuming elastic behavior for the composite and constituents there follows

$$\{\sigma\} = [C](\{\varepsilon\} - \{\varepsilon_0\}), \tag{6}$$

$$\{\sigma_\alpha\} = [C_\alpha](\{\varepsilon_\alpha\} - \{\varepsilon_{\alpha 0}\}), \tag{7}$$

$$\{\sigma_\beta\} = [C_\beta](\{\varepsilon_\beta\} - \{\varepsilon_{\beta 0}\}), \tag{8}$$

where $[C]$, $[C_\alpha]$, and $[C_\beta]$ represent material stiffness matrices and $\{\varepsilon_0\}$, $\{\varepsilon_{\alpha 0}\}$, and $\{\varepsilon_{\beta 0}\}$ are stress-free (assumed here to be thermal) strains. Let the thermal strains be defined as

$$\{\varepsilon_0\} = \theta\{\eta\}, \qquad \{\varepsilon_{\alpha 0}\} = \theta\{\eta_\alpha\}, \qquad \{\varepsilon_{\beta 0}\} = \theta\{\eta_\beta\},$$

where $\{\eta\}$ represents the coefficients of thermal expansion and θ is the relative temperature. Eqs. (4)–(8) can be combined to yield an expression for the constituent strain $\{\varepsilon_\alpha\}$ as a function of the composite strain given by

$$\{\varepsilon_\alpha\} = (\phi_\alpha[1] + \phi_\beta[A])^{-1}(\{\varepsilon\} - \theta\{a\}), \tag{9}$$

where

$$[A] = -\frac{\phi_\alpha}{\phi_\beta}([C] - [C_\beta])^{-1}([C] - [C_\alpha]), \tag{10}$$

[1] is the identity matrix, and

$$\{a\} = ([C] - [C_\beta])^{-1}([C]\{\eta\} - \phi_\beta[C_\beta]\{\eta_\beta\} - \phi_\alpha[C_\alpha]\{\eta_\alpha\}). \tag{11}$$

Given $\{\varepsilon_\alpha\}$ from Eq. (9), Eq. (5) yields an expression for $\{\varepsilon_\beta\}$ as:

$$\{\varepsilon_\beta\} = \frac{1}{\phi_\beta}(\{\varepsilon\} - \phi_\alpha\{\varepsilon_\alpha\}). \tag{12}$$

Eqs. (9) and (12) allow phase-averaged constituent strains to be calculated from composite strains at any point in a structural finite element model. Constituent stresses can be calculated using Eqs. (7) and (8).

Any application of the above decomposition requires a connection between composite properties and the properties of individual constituents. Typically $[C_\alpha]$, $[C_\beta]$, $\{\eta_\alpha\}$, and $\{\eta_\beta\}$ are assumed known material properties of the constituents. Composite terms, $[C]$ and $\{\eta\}$, can be developed from finite element micromechanical models using the constituent values as input.

3. Extension to a three-constituent multicontinuum theory

An MCT analysis of a woven fabric composite material poses substantially greater difficulties than a unidirectional composite in that there are now three constituents to deal with. Specifically, we treat the weave as a three-constituent composite composed of warp fiber bundles (α), fill fiber bundles (β), and pure matrix pockets (γ). The addition of a third constituent results in an indeterminate set of equations based on the traditional decomposition put forth by Hill. The extension of an MCT decomposition to woven fabrics is summarized below and may be found in Key et al. [5].

Fig. 3 illustrates a generic three-constituent continuum point consisting of constituents α, β, and γ. The introduction of the third constituent, γ, to the continuum adds the following constitutive relation to the previously described system of equations:

$$\{\sigma_\gamma\} = [C_\gamma](\{\varepsilon_\gamma\} - \{\varepsilon_{\gamma 0}\}). \tag{13}$$

With the addition of the third constituent, γ, and noting the previous development, we have introduced one additional equation, (13), and two unknowns given by $\{\sigma_\gamma\}$ and $\{\varepsilon_\gamma\}$. This leads to a set of equations that is indeterminate.

To eliminate the indeterminacy introduced by a third constituent, a treed approach is utilized as outlined in Fig. 4. In this approach, we first combine the warp fiber bundle (α) and fill fiber bundle (β) constituents into a single constituent denoted by αβ. This combination allows the previously indeterminate set of equations to be reduced to a set of branched two-constituent problems, each composed of determinant sets of equations. The first branch of the treed structure consists of constituents αβ and γ, with unknowns $\{\sigma_{\alpha\beta}\}$, $\{\varepsilon_{\alpha\beta}\}$, $\{\sigma_\gamma\}$, and $\{\varepsilon_\gamma\}$. For this first branch of the three-constituent theory, Eqs. (9)–(12) are modified as

$$\{\varepsilon_\gamma\} = (\phi_\gamma[I] + \phi_{\alpha\beta}[A])^{-1}(\{\varepsilon\} - \theta\{a\}), \tag{14}$$

where

$$[A] = -\frac{\phi_\gamma}{\phi_{\alpha\beta}}([C] - [C_{\alpha\beta}])^{-1}([C] - [C_\gamma]), \tag{15}$$

$$\{a\} = ([C] - [C_{\alpha\beta}])^{-1}([C]\{\eta\} - \phi_{\alpha\beta}[C_{\alpha\beta}]\{\eta_{\alpha\beta}\} - \phi_\gamma[C_\gamma]\{\eta_\gamma\}), \tag{16}$$

and

$$\{\varepsilon_{\alpha\beta}\} = \frac{1}{\phi_{\alpha\beta}}(\{\varepsilon\} - \phi_\gamma\{\varepsilon_\gamma\}). \tag{17}$$

The constitutive equation for the αβ constituent assumes the form:

$$\{\sigma_{\alpha\beta}\} = [C_{\alpha\beta}](\{\varepsilon_{\alpha\beta}\} - \{\varepsilon_{\alpha\beta 0}\}). \tag{18}$$

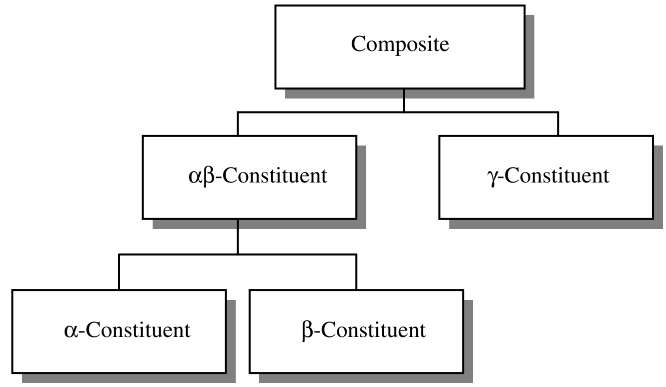

Fig. 4. Three constituent tree structure.

Once $\{\sigma_{\alpha\beta}\}$ and $\{\varepsilon_{\alpha\beta}\}$ are calculated in the first branch of the theory, the αβ constituent can then be viewed as the composite for the second branch of the tree, where α and β are its respective constituents. The fundamental strain relations given in Eqs. (9)–(12) are again modified as

$$\{\varepsilon_\alpha\} = (\phi_\alpha[I] + \phi_\beta[A])^{-1}(\{\varepsilon_{\alpha\beta}\} - \theta\{a\}), \tag{19}$$

where

$$[A] = -\frac{\phi_\alpha}{\phi_\beta}([C_{\alpha\beta}] - [C_\beta])^{-1}([C_{\alpha\beta}] - [C_\alpha]), \tag{20}$$

$$\{a\} = ([C_{\alpha\beta}] - [C_\beta])^{-1}([C_{\alpha\beta}]\{\eta_{\alpha\beta}\} - \phi_\beta[C_\beta]\{\eta_\beta\} - \phi_\alpha[C_\alpha]\{\eta_\alpha\}), \tag{21}$$

and

$$\{\varepsilon_\beta\} = \frac{1}{\phi_\beta}(\{\varepsilon_{\alpha\beta}\} - \phi_\alpha\{\varepsilon_\alpha\}). \tag{22}$$

A subtle but important point in Eqs. (19)–(22) is that the volume fractions ϕ_α and ϕ_β represent the volume of constituents α and β relative to the volume of the αβ constituent.

Any application of the proposed three-constituent decomposition requires one to determine the material stiffness matrix $[C_{\alpha\beta}]$ as well as the coefficients of thermal

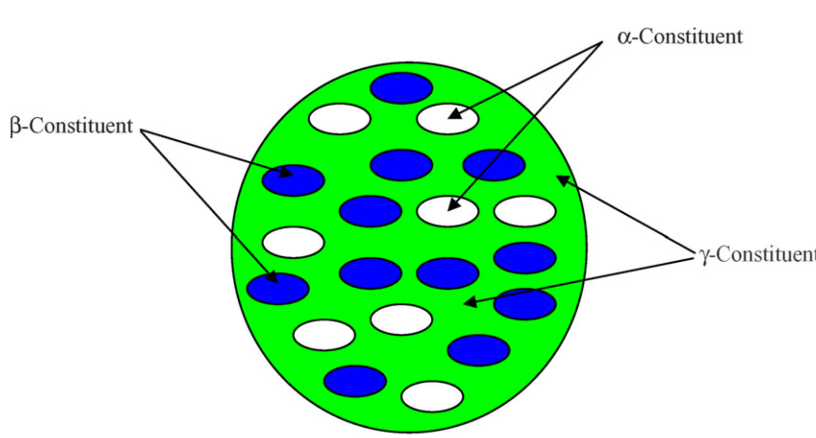

Fig. 3. Generic three constituent 'continuum point'.

expansion $\{\eta_{\alpha\beta}\}$. To determine these material properties we again rely on the finite element micromechanics model. A judicious selection of mechanical load cases for the composite allows one to induce specific stress states within the $\alpha\beta$ constituent that lead to straightforward calculations of the material properties. Once $[C_{\alpha\beta}]$ is known, $\{\eta_{\alpha\beta}\}$ may be determined by applying a thermal load to the micromechanics model. Upon volume averaging the appropriate strain fields for the thermal load, Eq. (19) may be used to determine the vector $\{a\}$. Substituting $\{a\}$ into Eq. (21) allows one to compute $\{\eta_{\alpha\beta}\}$ directly.

The geometry of a balanced plain weave composite presents some difficulties associated with the proposed three-constituent decomposition outlined above. In particular, singular matrices are encountered in the second decomposition, thereby preventing required matrix inversions. The traditional decomposition is altered by condensing out appropriate stress/strain terms that produce the singular matrices. The reader is referred to Key et al. [5] for details.

Finally, when developing a continuum damage model for the weave we note that damage in the warp and fill fiber bundles is assumed to occur in the matrix constituent within the bundle. Accessing constituent information within a fiber bundle requires one to essentially nest the MCT decompositions. Hence, the three-constituent MCT decomposition is executed first to generate stress/strain fields in the fiber bundles. The two constituent MCT decomposition is then executed to determine the matrix stress and strain fields within a fiber bundle. Finally, given access to the matrix stress/strain fields within a fiber bundle in the weave, we develop a continuum damage model in an effort to predict ultimate composite failure while capturing the inelastic stress–strain response.

4. Failure criteria and material degradation

In this section, we develop two separate progressive failure models for a plain weave composite. Progressive failure analyses using constituent based failure criteria have been previously implemented for unidirectional composite materials with good success [1–3]. However, we emphasize that MCT is not a failure criterion. Rather, it is a mechanism to introduce constituent level information into a failure analysis. Indeed, any user of MCT may implement a failure criterion of their own choosing.

4.1. Stress based failure with instantaneous material property degradation

The first modeling approach used to predict progressive failure within the woven fabric material is a stress based criterion with instantaneous (binary) degradation. Although, the bundles within the woven fabric microstructure have some degree of undulation to them, for this analysis we assume that the undulation is small and therefore we can assume that the bundles may be treated locally as being transversely isotropic. Noting this we assume a simplified quadratic stress interactive failure criteria for the bundles based on the work of Mayes [1] given by:

$$K_1 I_1^2 + K_2 I_2^2 + K_3 I_3 + K_4 I_4 = 1. \tag{23}$$

The transversely isotropic invariants in Eq. (23) may be expressed as:

$$\begin{aligned} I_1 &= \sigma_{11}, \\ I_2 &= \sigma_{22} + \sigma_{33}, \\ I_3 &= \sigma_{22}^2 + \sigma_{33}^2 + 2\sigma_{23}^2, \\ I_4 &= \sigma_{12}^2 + \sigma_{13}^2, \\ I_5 &= \sigma_{22}\sigma_{12}^2 + \sigma_{33}\sigma_{13}^2 + 2\sigma_{12}\sigma_{13}\sigma_{23}, \end{aligned} \tag{24}$$

where the x_1 direction represents the fiber direction. The failure criterion of Eq. (23) is applied to both the warp and fill fiber bundles in the woven fabric microstructure to determine the constituent mode or modes of failure within each bundle.

Following the reasoning of Mayes [1], we make the following observations about failure within the fiber bundles. Longitudinal fiber bundle failure is controlled by fiber failure within the bundle, while transverse failure is controlled by matrix failure within the bundle. Therefore, we define a longitudinal (fiber) failure criterion and a transverse (matrix) failure criterion for each of the fiber bundles as

$${}^{\pm}K_{1f} I_{1f}^2 + K_{4f} I_{4f} = 1, \tag{25}$$

and

$${}^{\pm}K_{2m} I_{2m}^2 + K_{3m} I_{3m} + K_{4m} I_{4m} = 1. \tag{26}$$

We reiterate that for this stress based instantaneous degradation modeling approach, the fiber bundles are treated as individual constituents within the woven fabric microstructure. Therefore, the coefficients and invariants given in Eqs. (24)–(26) are properties of the unidirectional fiber bundles.

In direct correlation with the derivations of Mayes, the coefficients for the longitudinal (fiber) failure criteria of Eq. (25) are given by

$${}^{\pm}K_1^{L} = \frac{1}{({}^{\pm}S_{11})^2} \tag{27}$$

and

$$K_4^{L} = \frac{1}{(S_{12}^{L})^2}. \tag{28}$$

Likewise, the coefficients for the transverse (matrix) failure criteria of Eq. (26) are given by

$$\pm K_2^{T} = \frac{1}{({}^{\pm}S_{22})}\left(1 - \frac{({}^{\pm}S_{22})^2}{2(S_{23})^2}\right), \tag{29}$$

$$K_3^{T} = \frac{1}{2(S_{23})^2}, \tag{30}$$

and

$$K_4^{T} = \frac{1}{(S_{12}^{T})^2}. \tag{31}$$

The in-plane shear coefficient for longitudinal (fiber) failure (S_{12}^{L}) given in Eq. (28) differs from the coefficient for transverse (matrix) failure (S_{12}^{T}) given in Eq. (31). These different shear strength coefficients differentiate between longitudinal (fiber) bundle failure caused by shear stresses and transverse (matrix) bundle failure caused by shear stresses.

4.2. Damage based failure with continuous material property degradation

In a second method of modeling the failure response of woven fabric composites we introduce a continuous damage approach which utilizes a damage evolution for the matrix constituent within a bundle. The damage evolution is motivated by a one-dimensional damage model that exhibits the characteristics of stress and time dependence based on the kinetic theory of fracture.

Kinetic theory is centered around bond rupture at the molecular level in a material. Bond rupture occurs at the molecular level and manifests itself in the form of submicrocracks. As loading continues, these microcracks coalesce resulting in macroscopic failure. The evolution of microcracks under uniaxial stress is represented by the following differential equation [6]

$$\frac{dN(t)}{dt} = (N_T - N(t))K_b, \tag{32}$$

where N is the number of submicrocracks, N_T is a constant representing local 'hot spots' such as amorphous–crystalline interfaces, etc. and K_b is the reaction rate for material breakage given by:

$$K_b = \frac{1}{\tau_0} e^{(-(U-\gamma\sigma)/kTA)}. \tag{33}$$

In Eq. (33), τ_0 is the period of characteristic oscillation of atoms in a solid, k is Boltzmann's constant, A is Avogadro's number, T is the temperature, and U and γ are material constants.

Dividing Eq. (32) by N_r, where N_r represents the number of submicrocracks at rupture, the degree of damage can be represented on a scale of $0 < n < 1$, where $n = 0$ represents no damage and $n = 1$ represents macroscopic material failure. The resulting differential equation representing the degree of damage accumulation within a material is given by:

$$\frac{dn(t)}{dt} = (n_0 - n(t))K_b. \tag{34}$$

The extension of the one-dimensional kinetic theory damage model to three-dimensional stress states is achieved by introducing a second order continuum damage tensor, n_{ij}, given by:

$$n_{ij} = \begin{bmatrix} n_{11} & n_{12} & n_{13} \\ & n_{22} & n_{23} \\ \text{sym} & & n_{33} \end{bmatrix}. \tag{35}$$

The damage tensor components are assumed to satisfy the evolution equations given by

$$\frac{dn_{ij}(t)}{dt} = (n_0 - n_{ij}(t))K_{ijb}, \tag{36}$$

where

$$K_{ijb} = \frac{1}{\tau_0} e^{(-(R-\beta\sigma_{ij}^{m}))}. \tag{37}$$

In Eq. (37) R and β represent material constants.

In the above, we are associating damage in the composite with the corresponding stress component seen by the *matrix* material within each bundle. As a result, the damage tensor is symmetric due to the symmetry of the stress tensor. Within a finite element program, the degree of damage is calculated at every Gauss point. Once the degree of damage is known, the damage accumulation is used to control the degradation of elastic material properties.

4.3. Unidirectional composite

In order to develop a damage based failure criterion for the woven fabric microstructure it is first necessary to develop the damage model for a unidirectional composite due to the fact that the fiber bundles within the woven fabric are treated as unidirectional materials. For the transversely isotropic unidirectional composite, we assume that matrix damage accumulates in such a way that the composite and in situ matrix both remain transversely isotropic. Therefore, the damage is expressed in terms of the transverse isotropic damage invariants given by:

$$\begin{aligned} I_1 &= n_{11}, \\ I_2 &= n_{22} + n_{33}, \\ I_3 &= n_{22}^2 + n_{33}^2 + 2n_{23}^2, \\ I_4 &= n_{12}^2 + n_{13}^2, \\ I_5 &= n_{22}n_{12}^2 + n_{33}n_{13}^2 + 2n_{12}n_{13}n_{23}. \end{aligned} \tag{38}$$

As damage in the matrix material accumulates, the matrix properties are degraded based on the damage invariants of Eq. (38) to reflect reduced stiffness. The development of the material property degradation models are described in detail by Schumacher [7]. For brevity, only the functional forms of the degradation models are presented here. The form of the material degradation for G_{12}^{m} and G_{13}^{m} are a function of I_4 and are given by

$$G_{12}^{m'} = G_{12}^{m}(1 - \sqrt{I_4}), \tag{39}$$

and

$$G_{13}^{m'} = G_{13}^{m}(1 - \sqrt{I_4}), \tag{40}$$

where the primed (′) value denotes the degraded stiffness.

Motivation for the degradation models of Eqs. (39) and (40) is provided by recalling the fourth invariant from Eq. (38) which is a function of n_{12} and n_{13}. Hence, for the case of a longitudinal shear test where $\sigma_{13} \neq 0$, Eq. (40) becomes:

$$G_{13}^{m'} = G_{13}^{m}(1 - n_{13}). \tag{41}$$

Notice when $n_{13} = 0$, the material is undamaged, whereas $n_{13} = 1$ would effectively zero the shear modulus.

Now consider the damage components n_{22} and n_{33} caused by transverse tensile stresses. The damage accumulation is assumed to affect the matrix elastic properties E_{22}^{m}, E_{33}^{m} and, in addition, G_{23}^{m} by transverse isotropy. Similarly, in the case of transverse shear damage, n_{23} is assumed to affect the matrix elastic properties G_{23}^{m}, E_{22}^{m}, and E_{33}^{m}.

The assumed form of material degradation of E_{22}^{m}, E_{33}^{m}, and G_{23}^{m} is shown below where the material degradation is only dependent upon I_3, i.e.

$$E_{22}^{\mathrm{m}'} = E_{22}^{\mathrm{m}}\left(1 - \sqrt{\frac{I_3}{2}}\right). \tag{42}$$

$$E_{33}^{\mathrm{m}'} = E_{33}^{\mathrm{m}}\left(1 - \sqrt{\frac{I_3}{2}}\right). \tag{43}$$

and

$$G_{23}^{\mathrm{m}'} = G_{23}^{\mathrm{m}}\left(1 - \sqrt{\frac{I_3}{2}}\right). \tag{44}$$

Finally it is noted that the composite stiffness properties must be degraded in a manner consistent with the constituents. A detailed discussion of the functional degradation of the composite properties is provided by Schumacher [7]. However it is worth noting that, under a three-dimensional state of stress, the transverse tension and transverse shear composite elastic material properties are degraded simultaneously, thereby preserving transverse isotropy while taking into account the directional damage dependence.

4.4. Woven fabric composites

The previous damage model for unidirectional composites represents a critical component of the damage model for woven fabrics. Specifically, damage in the weave is attributed to matrix cracking occurring within the fiber bundles. Within the woven fabric composite, the fiber bundles are treated as unidirectional subcomposites.

The previously outlined approach requires the degree of *matrix* damage within the fill and warp bundles be determined. This task requires two additional MCT stress decomposition branches to be added to the open weave analysis, as shown in Fig. 5. Finally, a functional relationship, similar to that given previously for a unidirectional composite, between the fiber bundle damage and the composite properties must be generated. Again, the reader is referenced to Schumacher [7] for a detail discussion of the functional degradation schemes.

A final piece of the damage based modeling approach is the development of a macroscopic failure criterion. The damage interactive failure criterion developed is similar to the stress interactive failure criterion presented previously in the stress based instantaneous degradation model. Specifically, the matrix failure criterion is given by

$$A_2^{\mathrm{m}}(I_2^{\mathrm{m}})^2 + A_3^{\mathrm{m}} I_3^{\mathrm{m}} + A_4^{\mathrm{m}} I_4^{\mathrm{m}} = 1, \tag{45}$$

where the invariants are *matrix damage invariants* for the matrix constituent within the fiber bundles.

In Eq. (45), the coefficients $A_{\mathrm{i}}^{\mathrm{m}}$ are determined from unidirectional composite experimental stress–strain behavior in a manner identical to that presented by Mayes. Therefore, a transverse tension loading case is used to determine the coefficient A_2^{m}, as:

$$A_2^{\mathrm{m}} = \frac{1}{(F_{22}^{22\mathrm{m}} + F_{33}^{22\mathrm{m}})^2}\left(1 - \frac{(F_{22}^{22\mathrm{m}})^2 + (F_{33}^{22\mathrm{m}})^2}{2(F_{23}^{\mathrm{m}})^2}\right). \tag{46}$$

The coefficients $F_{jj}^{ii\mathrm{m}}$ represent the critical values of damage as determined from stress–strain experimental data. The double subscript notation is necessary to identify the damage term and the loading direction. In particular, $F_{33}^{22\mathrm{m}}$ represents damage in the 33 direction as the result of a stress in the 22 direction. Such notation is necessary because the stress state in the matrix material is fully three-dimensional under uniaxial composite stress. For the case of out-of-plane shear (transverse shear), the critical coefficient A_3^{m} is given by:

$$A_3^{\mathrm{m}} = \frac{1}{2(F_{23}^{\mathrm{m}})^2}. \tag{47}$$

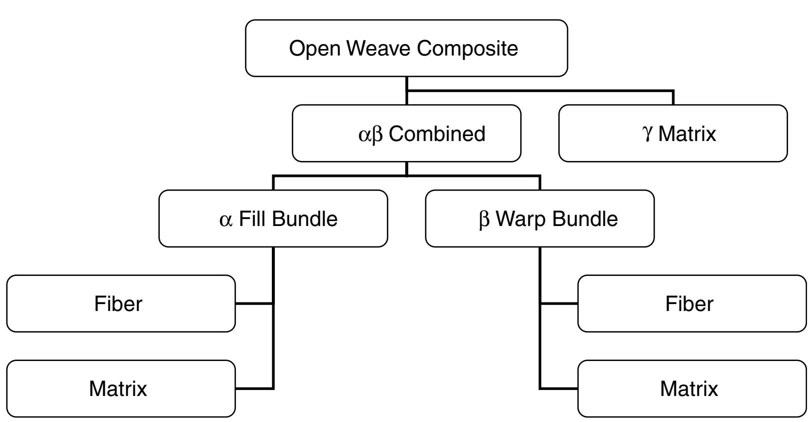

Fig. 5. Damage decomposition structure for a woven fabric.

In the case of in-plane shear (longitudinal shear), the calculation of the critical coefficient, A_4^m is given by:

$$A_4^m = \frac{1}{(F_{12}^m)^2}. \quad (48)$$

Once a damage based failure has occurred, all matrix properties with a fiber bundle are set to near zero values at the failed Gauss points.

The final piece of the damage based failure criterion is that the fiber failure criterion for this approach is identical to that for the stress based approach presented previously. This is due to the fact that although the matrix constituent commonly accumulates damage in the form of microcracks in a composite material, the fibers in these systems are assumed to fail instantaneously.

5. Comparison of analysis versus experiment

In this section, both the stress based instantaneous degradation and the damage based continuous degradation models are compared against experimental data for a plain weave composite material subjected to uniaxial tension and in-plane shear. Stress–strain curves and a biaxial failure envelope are used to qualify the ability of MCT to predict laminate level behavior for a woven fabric composite. The material system used for this study was an 18 oz. E-glass/vinylester (Dow 8084) system with an assumed fiber bundle volume fraction of 75% and an overall fiber volume fraction of 50%. The specimens for the experimental testing were fabricated by Seemann Composites, Inc., with their patented SCRIMP process. Elastic constants and failure parameters related to both modeling approaches are given in Tables 1–3.

5.1. Uniaxial tension

The first loading condition examined for the validation procedure was uniaxial tension of a $[0_6]_f$ laminate. Fig. 6 shows the experimental stress–strain response versus the MCT predicted stress–strain responses for both the stress based and damage based modeling approaches. This figure highlights the damage induced nonlinear behavior commonly seen in composite materials and the ability of the MCT constituent level progressive failure algorithm to capture these nonlinearities.

Table 3
Damage parameters for damaged based MCT

	Normal component	Shear component
Initial damage, n_0	1.58	1.58
Activation energy, U (J/mol)	1.17×10^5	8.48×10^4
Material constant, γ (MPa/mol)	1.0×10^{-3}	1.0×10^{-4}
τ_0 (s)	1.0×10^{-13}	1.0×10^{-13}
Ultimate damage (F)	0.216	0.86

For both of the MCT modeling approaches, failure initiates as matrix failure within the fiber bundles that are oriented transverse (warp bundle) to the loading direction. This initial failure point is labeled point A in both of the predicted stress–strain responses. The intermediate failure state of matrix failure within a fiber bundle (point A) is commonly observed during experimental testing. Fig. 1 illustrates this failure condition, where the milky regions seen in the dog-bone specimen are regions of matrix failure within the transverse fiber bundles.

As loading is increased in the analyses, the damage based MCT approach experiences matrix failure within the longitudinal fiber bundles (fill bundles) at point B before catastrophic specimen failure. This is in direct contrast to the stress based MCT approach, where matrix failure within the longitudinal fiber bundles (fill bundles) does not occur. For both methods, catastrophic failure results from a tensile fiber failure in the longitudinal bundles at point C. Both methods predict final failure within 3% of the experimental value.

5.2. In-plane shear

The second loading condition studied was in-plane shear of a $[0_6]_f$ laminate. To achieve this loading condition, an Iosipescu shear test was utilized [8]. Fig. 7 shows the experimental stress–strain response along with the MCT predicted stress–strain responses. In this figure, the stress based MCT approach predicts a linear stress–strain

Table 1
Composite and fiber bundle material properties for E-glass/8084 vinylester

	E11 (GPa)	E22 (GPa)	E33 (GPa)	υ12	υ13	υ23	G12 (GPa)	G13 (GPa)	G23 (GPa)
Composite	26.9	26.9	14.5	0.138	0.0278	0.0278	5.54	7.93	7.93
Fiber bundles	55.3	22.6	22.6	0.232	0.238	0.286	9.19	9.36	8.80

Fiber bundle properties are given in the local coordinate system (fiber direction: x_1).

Table 2
Fiber bundle ultimate strengths

	$+S_{11}$ (MPa)	$-S_{11}$ (MPa)	$+S_{22}$ (MPa)	$-S_{22}$ (MPa)	S_{12}^{L} (MPa)	S_{12}^{T} (MPa)	S_{23} (MPa)
Fiber bundle ultimate strengths	1135	−1054	180.0	−160.1	102.9	50.69	210.0

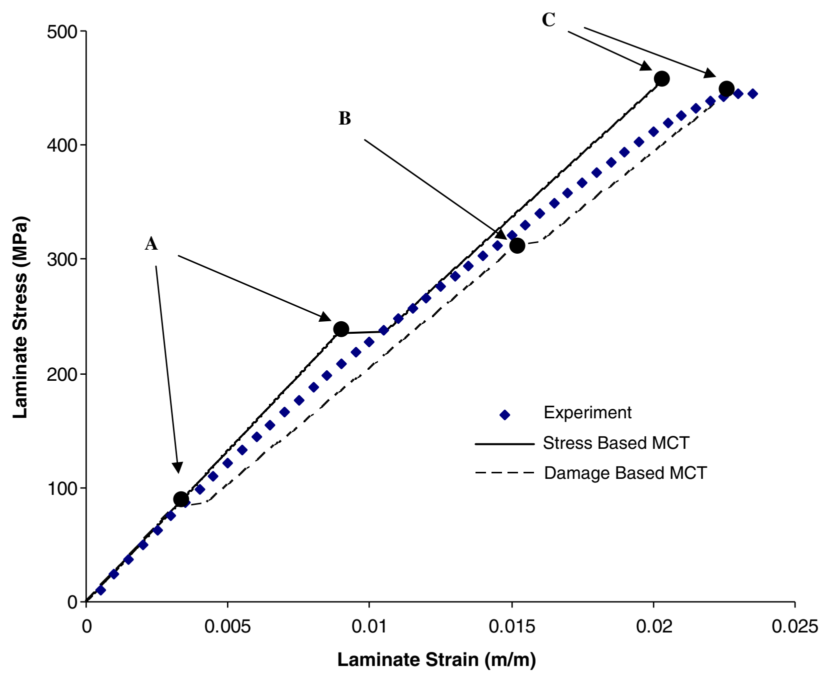

Fig. 6. E-glass/vinylester woven fabric laminate under uniaxial tension.

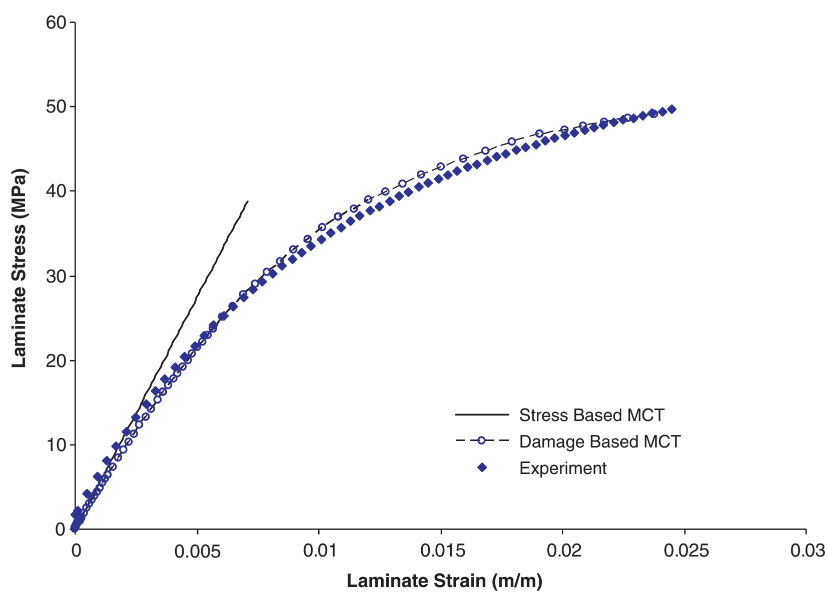

Fig. 7. E-glass/vinylester woven fabric laminate under in-plane shear.

response while the damage based modeling captures the nonlinear behavior of the material. The ability of the damage based MCT approach to capture the highly nonlinear behavior of composite materials under shear loading conditions is a vital capability for an analysis tool. For the damage based MCT, the predicted ultimate strength is within 2% of the experimental ultimate value while the stress based approach under predicts the stress and strain by 22 and 71%, respectively. Both catastrophic failure points for the MCT models are caused by failure of the matrix constituent within both the fill and warp bundles at simultaneous instances.

5.3. Biaxial loading

In further studying the MCT failure prediction with the stress based approach, a two-dimensional failure envelope was generated from biaxial tests on thickness-tapered cruciform specimens with a layup sequence of $[0/90]_s$. The thickness-tapered cruciform specimens were tested using a

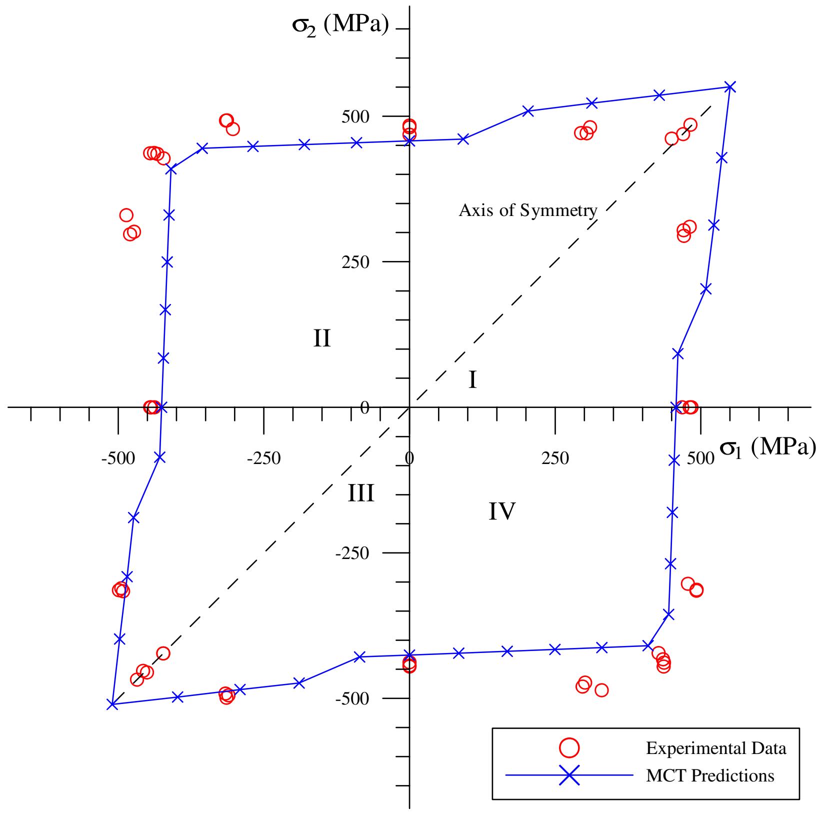

Fig. 8. E-glass/vinylester woven fabric biaxial failure envelope [10].

triaxial test apparatus [9–11]. The triaxial test apparatus is capable of generating any combination of tensile or compressive stresses in σ_1:σ_2 stress space. All of the tests for this work were load controlled with a minimum of three tests ran for each of the following applied x/y load ratios: 1/1, 2/1, 1/0, 2/−1, 1/−1, 1/−2, −1/0, −1/−2, and −1/−1.

Fig. 8 presents both the experimental and MCT predicted biaxial failure envelopes for the woven fabric laminate [10]. The analytical MCT predictions for ultimate strength are in excellent agreement with the experimentally determined data.

6. Summary

The ability to access woven fabric composite material failure at the constituent level opens a new window of understanding related to the response and behavior of these materials. Most often failure in these materials is not caused by one single catastrophic event, but rather the accumulation of multiple intermediate failure events. In this work, two separate failure criteria and degradation methods were studied and compared against experimental data. The stress based instantaneous degradation model does a good job of predicting the stress–strain response for uniaxial tension. However, it does a poor job under in-plane shear where a large amount of damage induced nonlinearity is seen. In contrast, the damage based MCT approach does a good job of predicting the damage induced nonlinear stress–strain response for both the normal and shear loading. This improvement is largely due to the continuous degradation scheme used for the matrix stiffness properties within both the fill and warp fiber bundles.

The authors are pleased with the results of this work, especially when considering all of the complexities such as microstructure geometry, material constants, etc. that are associated with the modeling woven fabric composites. Finally, the work contained herein is not limited in application to plain weave composites and can be expanded to handle various other woven fabric architectures such as satin weaves, harness weaves, etc.

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